,,Three Essays Inferring Prospective and Retrospective Information Based on Options Trading Activities and a New Theoretical Approach on Multivariate Subordination of Lévy Processes"

Dissertation for the Faculty of Economics, Business Administration and Information Technology of the University of Zurich

to achieve the title of

Doctor of Philosophy

in Banking & Finance

presented by

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorises the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

Zurich, 27.10.2010

Chairman of the Doctoral Committee: Prof. Dr. Dieter Pfaff

To my parents: to my mother, who taught me the true meaning of love, and to my father, who was taken from us far too early .

Zurich, October 2010

Acknowledgments

This thesis has benefited from the excellent environment I found at the Swiss Banking Institute at the University of Zurich. My warmest thanks go to Professor Marc Chesney, my thesis supervisor, for his outstanding guidance and comments during my doctoral studies. I would also like to thank professors Markus Leippold, Loriano Mancini and Fabio Trojani for their help with and contribution to our joint papers.

The generous financial support from the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK), the Swiss Finance Institute (SFI) and research instruments at the University of Zurich are gratefully acknowledged.

Finally, I would like to thank my family and friends for their support during the dissertation process. In particular, my deepest gratitude goes to my mother, my sister, my brother and their families and to Marco who have always encouraged me in my pursuits.

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Part I

Introduction

Executive Summary

It is a well-known and extensively studied phenomenon that market participants in possession of private information actively trade in the stock market. Less known and researched are informed trading activities in the options market. Nevertheless, various incentives such as low initial capital, high financial leverage and discreetness offered by options markets could induce traders with privileged information to trade in options rather than in the underlying asset.

The first chapter of this thesis aims therefore to detect informed trading activities based on put options. It contributes to the existing literature in two ways: firstly, it studies informed trading activities in options rather than stock markets, and secondly, it provides a statistical method to detect informed trades in option contracts. An option trade is identified as informed when it is characterized by an unusually large increment in open interest and volume, induces large gains, and is not hedged in the stock market. As an empirical application of this new detection procedure, each put option contract on 14 companies traded on the Chicago Board Options Exchange during the period 1996-2006 are analyzed. Three European companies with options traded on the Eurex are also considered. The method detects several informed trades which can be connected to one of the three following events: merger and acquisition announcements, quarterly financial/earnings related statements, and the terrorist attacks of September 11th, 2001.

In the second chapter of this thesis, the model is extended to call options, and option trading strategies with underlying financial and insurance institutions strongly affected by the recent financial crisis are analyzed. Three various options markets are explored: the Chicago Board Options Exchange (CBOE), with companies such as AIG, Lehman

Brothers, Bear Stearns, Fannie Mae and Freddie Mac, among others; Eurex (Zurich and Frankfurt), with United Bank of Switzerland (UBS), Credit Suisse Group and Deutsche Bank; and Euronext (Paris and London), with Société Générale, BNP Paribas and HSBC. The empirical findings suggest that periods leading up to key events such as the takeovers of AIG and Fannie Mae/Freddie Mac, the collapse of Bear Stearns Corporation and public announcements relating to large losses/writedowns are preceded by profitable trading activities in put and call options.

Motivated by the empirical findings of the two first chapters, in the third chapter the (ex-ante) informational content of large changes in open interest is studied when the linkage between option market variables and subsequent price movements in the underlying stock are investigated. A daily statistic which measures the imbalance between newly issued puts and calls is defined. Conditional on this, the cumulative distribution of several indicators of future market activities is estimated. Differences between the unconditional and conditional distribution are used as a measure for the predictive power of the daily statistic. Chapter three empirically shows that whenever the imbalance between newly issued puts and calls takes extreme values, the conditional distribution exhibits significant changes with respect to its unconditional counterpart: when the number of newly issued put options is large compared to the number of newly issued call options, the conditional distribution functions of the idiosyncratic return noise process becomes heavier on the left side and a large drop in the underlying stock is more likely to follow. In the opposite scenario, when the statistic exhibits a large imbalance in favor of call options, the idiosyncratic return noise tends to be higher than after calm days. Those findings confirm the informational content of large daily changes in open interest.

The fourth and last chapter of this thesis moves to a purely theoretical setting and proposes a new multivariate time-change technique for stochastic processes used in the modeling of financial assets. Multivariate returns of financial assets feature a number of important characteristics. First, they can jump, leading to multivariate non-normal behavior. Second, their volatilities and correlations can vary stochastically over time. Third, returns co-move with their volatilities and correlations, often negatively for equities. Fourth, returns, volatilities and correlations can co-jump, leading to self-exciting market behavior. A general family of multivariate time changed Lévy processes that can simultaneously address these issues using a new class of multivariate time changes based on a matrix subordination approach is presented in this chapter. The framework includes as special cases many models in the literature, gives rise to a variety of new multivariate models, and is likewise simple to apply using the characteristic function methodology also used in the univariate context.

Part II

Research Papers

1

Detecting Informed Trading Activities in the Options Markets

Marc Chesney, Remo Crameri, Loriano Mancini

Summary. This paper presents a new method to detect informed trading activities in the options markets. An option trade is identified as informed when it is characterized by an unusual large increment in open interest and volume, induces large gains, and is not hedged in the stock market. For the period 1996–2006, each put option contract on 14 companies traded in the Chicago Board Options Exchange is analyzed. Three European companies with options traded on the Eurex are also considered. Our method detects several informed trades which can be associated to one of the following three events: merger and acquisition announcements, quarterly financial/earning related statements, and the terrorist attacks of September 11th.

Keywords: Put Options, Open Interest, Informed trading

JEL Classification: G12, G13, G14, G17, G34, C61, C65

1.1 Introduction

Informed trading activities in stock markets have been extensively studied in Finance. Various researchers have investigated the fundamental economic question of how new information gets incorporated into asset prices, how various frictions induced by trading mechanism impact this process, how informed traders should implement their trading strategies optimally to profit from their private signals, and other related aspects; e.g. [38], [21], [17], [20], [23], [25], [26], [6].

Our paper contributes to this literature in two directions: it studies informed trading activities in option rather than stock markets, and it provides a statistical method to detect informed trades in option contracts. Various incentives such as low initial capital, high financial leverage and discreetness offered by options market can induce traders with privileged information to trade in options rather than in the underlying asset. Unlike the stock market, options trading can involve the creation of new positions whenever the parties underwrite new contracts, increasing therefore the open interest (i.e. total number of existing option contracts on a given day). This paper shows that certain changes in open interest can reveal the information content of those specific trades.

From a legal point of view this study does not constitute proof per se of such activities. Legal proof would require trader identity and their motivations, information which is not contained in our database. Therefore, whenever we refer to informed trading activities, we think of *suspicious* trading activities.

According to our method, an option trade is identified as informed when it is characterized by a statistically large increment in open interest and volume, induces large returns and gains, and is not hedged in the stock market. Specifically, for each option the increment in open interest is compared to its daily volume to check whether or not this transaction can be classified as unusual. If so, the corresponding return and gain are calculated over various horizons. When the return and gain are statistically important, the probability that the option trade is not delta hedged is calculated. When this probability is sufficiently low, the option trade is identified as informed. This method is applied to each put option contract on 14 companies in various business sectors traded in the Chicago Board Options Exchange from January 1996 to April 2006 analyzing approximately 1.5 million of option contracts. In total 37 transactions are identified as informed trades: 6 occurring in the days leading up to merger and acquisition (M&A) announcements, 14 before quarterly financial/earnings related statements, 13 related to the terrorist attacks of September 11th, and 4 which could not be identified. For example four informed trades surrounding M&A announcements are detected in the airline sector. Two of them involved put options on American Airlines and United Airlines stock traded on May 10th and 11th, 2000, namely two weeks before UAL's acquisition of US Airways was announced. These trades generated a total gain of almost $3 million.[1] Another informed trade in a put option with underlying Delta Air Lines occurred a few weeks before the public announcement on January 21st, 2003 of the planned alliance among Delta, Northwest and Continental. In this case the total gain was more than $1 million. As noted in e.g. [28] and [7], takeover announcements are ideal events for studying information discovery in the security price formation process. Whereas trades made before scheduled announcements might be based on speculative bets, takeover announcements are not planned and trades prior to such events are likely to be started by traders who possess private information as detected by our method. Other detected informed trades can be associated to announcements related to drops in sales, production scale backs, and earnings shortfalls. For example three informed trades on put options with underlying Philip Morris stock are detected a few days before three separate legal cases against the company seeking a total amount of more than $50 million in damages for smokers' deaths and inoperable lung cancer. The corresponding gains in put options amounted to more than $10 million.

Our method is also applied to each put option on Swiss Re, Munich Re and EADS traded on EUREX from January 1999 to January 2008. Informed option trades on Swiss Re and Munich Re—the world's two largest reinsurers—are detected in the days leading up to the terrorist attacks on September 11th. Liabilities for the two companies were estimated to be in the amount of billions of dollars a few days after the attacks inducing large drops of stock prices and net gains in those transactions of more than €11.4 million. In the case of EADS, the parent of plane maker Airbus, six informed option trades are identified between April and June 2006. These trades precede the June 14th, 2006 announcement

[1] As reported in the New York Times edition of May 25th, 2000, AMR was considered the company most threatened by the merger, explaining therefore the 17% drop in its stock in the days after the public announcement. According to James Goodwin, chairman and chief executive of UAL, two major hurdles would challenge UAL: "the first is to get US Airways shareholders to approve this transaction. [The second] is the regulatory work, which revolves around the Department of Transportation, the Department of Justice and the European Union". The skepticism on Wall Street was immediately reflected on UAL shares which declined $7.19 to $53.19 on the announcement day.

that deliveries of the superjumbo jet A380 would be delayed by a further six months period, causing a 26% fall in the underlying stock, and a total gain of €7.5 million in these option trades.

The paper is organized as follows. Section 1.2 reviews related literature on informed trading. Section 1.3 introduces our methodology to detect option informed trades. Section 1.4 describes the database. Section 1.5 presents the empirical results. Section 1.6 concludes.

1.2 Related literature

This paper is mainly related to two strands of literature dealing with informed trading activities and linkages of information between option and stock markets. Analysis of informed trades has typically focused on specific events such as stock and option trading prior to M&A announcements (e.g. [28], [27] and [7]), asset returns around quarterly earnings announcements (e.g. [32], [1], [41], [43] and [12]), or option trades in the days leading up to the terrorist attacks of September 11th ([40]). Our paper contributes to this literature in several ways. First, it does not focus on a single type of event but rather analyzes a long time period (more than ten years of daily and intraday data) uncovering various kinds of informed trading activities in different occasions. The case of EADS will be considered as an example. Second, previous papers use typically regression models in which the underlying stock return is the dependent variable and option variables are explanatory variables. We use a different, nonparametric approach. Option trades are identified as informed when they are statistically unusual according to the empirical probability of that event. Third, a novel feature in our approach is that it takes into account the hedging dimension. Option trades which are subsequently hedged should not be classified as informed trades. Fourth, we compute realized returns and gains from informed option trades quantifying the importance of such trades. Our methodology has some similarities to that of [40], such as using open interest to detect informed trading. However, there are also important differences concerning the data, method and aims. For example Poteshman focuses mainly on the airline sector and suspicious trading activities in the days leading up to the terrorist attacks of September 11th, but does not consider the potential hedging demand and uses a quantile regression approach. We perform a more general analysis, considering different sectors and events, and use a different approach. [21] and [18] develop the probability of information-based trading (PIN). This method has been mainly applied to detect informed trades in stock markets as for e.g. in [17], [19] and [45].

The second strand of literature investigates the linkage and information flow between options and stock markets; e.g. [13], [44], [16], [36], [20], [8], [39], [29], and [14]. In particular [20] introduce an equilibrium model where informed investors decide endogenously whether to trade in the stock and the option market in a "pooling equilibrium" and [39] provide empirical evidence of this equilibrium analyzing put-call ratios. Overall this research indicates that signed option volumes have an impact on future underlying asset price dynamics. [15] show that deviations from put-call parity contain information about future stock returns. Our goal is different. We aim at identifying the arrival of single informed trade in the option market for e.g. as soon as it takes place. Our findings suggest that informed trades detected by our procedure are not reflected into stock prices until the event occurs. Our paper is also related to the detection of insider trades, the latter being a subclass of informed trades; e.g. [37], [4], [34], [33] and [12]. Our empirical results show that option markets are profitable for informed traders suggesting that informed traders might consider options as superior trading vehicles; e.g. [3], [2], [42], [10], [35], [31] and [9].

[11] forecast asset crashes using shares trading volume. [5] emphasize the role of transaction volume as a tool for technical analysis. We complement these works by showing that certain increments in open interest have predictive power for future drops in the underlying stock. [46] studies information trading as well.

1.3 Detecting option informed trading activity

An informed trade in put options is defined as follows: C_1) an aggressive trade in an option contract, C_2) which is made a few days before the occurrence of a specific event and generates large gains in the following days, and C_3) the position is not hedged in the stock market and not used for hedging purposes. These three characteristics, $C_i, i = 1, 2, 3$, lead to the following method to detect informed trading activities: first on each day the put option contract with largest increment in open interest relative to its volume is identified, then the rate of return and dollar gain generated by this transaction are calculated, and finally it is studied whether hedging demands were at the origin of the trades. Options trades which are delta hedged are not regarded as informed trades. Below we describe in detail and apply this method to a large dataset of American put option trades. The method could be easily applied to call option trades as well.

Informed traders can obviously undertake a large variety of trading activities for example with various degrees of complexity to split their orders, jam the signals, etc. In this paper we restrict our attention to the economically sensible informed trade characterized by $C_i, i = 1, 2, 3$, above, that can be identified using available databases as we will see below.

1.3.1 The first criterion: Increment in open interest relative to volume

For every put option k available at day t we compute the difference $\Delta OI_t^k := OI_t^k - OI_{t-1}^k$, where OI_t^k is its open interest at day t and $:=$ means defined as. In the case that the option does not exist at time $t-1$, its open interest is set to zero. Since we are interested in unusual transactions, only the option with the largest increment in open interest is considered

$$X_t := \max_{k \in K_t} \Delta OI_t^k \tag{1.1}$$

where K_t is the set of all put options available at day t. The main motivation for considering increments in open interests is the following. Large volumes do not necessarily imply that large buy orders are executed because the same put option could be traded several times during the day. In contrast large increments in open interest are originated by large buy orders. These increments also imply that other long investors are unwilling to close their positions forcing the market maker to issue new put options. Let V_t denote the trading volume corresponding to the put option selected in (2.1). We focus on transactions for which the corresponding volume almost coincides with the increment in open interest. The positive difference $Z_t := (V_t - X_t)$ provides a measure of how often the newly issued options are exchanged: the smaller the Z_t, the less the new options are traded during the day on which they are created. In that case the originator of such transactions is not interested in intraday speculations but has reasons for keeping her position for a longer period possibly waiting for the realization of future events.

This first criterion already allows us to identify single transactions as potential candidates for informed trading activities. Let q_t denote the *ex-ante* joint historical probability of observing larger increment X_t in open interest and lower values of Z_t

$$q_t := \mathbb{P}[X \geq X_t, Z \leq Z_t] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{X_i \geq X_t, Z_i \leq Z_t\}}$$

where N represents the length of the estimation window, e.g. $N = 500$ days, and $\mathbf{1}_{\{A\}}$ is the indicator function of event A. By construction, low values of q_t suggest that these transactions were unusual. For example when $q_t = 1/N$, it means that what occurred on day t has no precedents in the previous two years.

1.3.2 The second criterion: Relative return and realized gain

The second criterion takes into consideration the ex-post relative returns and realized gains from transactions with a low ex-ante probability q_t. For each day t the trade with the largest increment in open interest is considered. Let R_t denote the maximum return generated in the following two trading weeks

$$R_t := \max_{j=1,\ldots,10} \frac{P_{t+j} - P_t}{P_t} \tag{1.2}$$

where P_t denotes the price of the selected option at day t. When R_t is unusually high, an unusual event occurs during the two trading weeks.

For the computation of realized gains, only the number of exercised options is considered. This can be done using decrements in open interest. Whenever the daily change in open interest of a specific option k, ΔOI_t^k, is negative, at least an amount of $|\Delta OI_t^k|$ options were exercised.[2] In the following we omit the superscript k and whenever we refer to a specific option we mean the one which was selected because of its largest increment in open interest and volume, i.e. lowest ex-ante probability q_t. It is generally more profitable to sell rather than exercise options but the OptionMetrics database used for our analysis does not provide information on that. Given our definition of informed trade, however, it is likely that on the event day the drop in the stock price is large enough to reach the exercise region. In the following we restrict our analysis to profits generated only through exercise. Hence our findings should be interpreted in a conservative manner.

Let G_t denote the corresponding cumulative gains achieved through the exercise of options

[2] The creation of new positions (which increases open interest), and the exercise of already existing options (which decreases open interest), can off-set each other so that a constant level of open interest does not necessarily mean that no options were exercised. In the database used for our analysis, OptionMetrics, the exercise of options can only be identified using the decrement in open interest which is a lower bound for the actual number of exercised options.

$$G_t := \sum_{\tilde{t}=t+1}^{\tau_t} [(K - S_{\tilde{t}})^+ - P_t] \cdot (-\Delta OI_{\tilde{t}}) \cdot \mathbf{1}_{\{\Delta OI_{\tilde{t}}<0\}}$$

where τ_t is such that $t < \tau_t \leq T$, with T being the maturity of the selected option. If the put options were optimally exercised (i.e. when the underlying asset $S_{\tilde{t}}$ is in the stopping region), the payoff $(K - S_{\tilde{t}})^+$ corresponds to the price of the option at time $\tilde{t}$. In principle the cumulative gains G_t could be calculated for every $\tau_t \leq T$. This has however the disadvantage that G_t can include gains which are realized through the exercise of options which were issued before time t.[3] Therefore time τ_t is defined as follows

$$\tau_t^* := \arg \max_{l \in \{t+1,\dots,T\}} \left\{ \sum_{\tilde{t}=t+1}^{l} (-\Delta OI_{\tilde{t}}) \cdot \mathbf{1}_{\{\Delta OI_{\tilde{t}}<0\}} \leq X_t \right\}$$

$$\tau_t := \min(\tau_t^*, 30)$$

giving the informed trader no more than 30 days to collect her gains. In general in the curly brackets the sum of negative decrements till time τ_t will be smaller than the observed increment X_t. In that case, we will add to G_t the gains realized through the fraction of the next decrement in open interest. Hence the sum of all negative decrements in open interest considered will be exactly equal to the increment X_t. Calculating G_t for each day t and each option in our database provides information on whether or not option trades with a low ex-ante probability q_t generate large gains through exercise. Using the maximal return R_t in (2.3) the ex-post joint historical probability p_t of the event $\{X_t, Z_t, R_t\}$ is

$$p_t := \mathbb{P}[X \geq X_t, Z \leq Z_t, R \geq R_t] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{X_i \geq X_t, Z_i \leq Z_t, R_i \geq R_t\}}.$$

The empirical probability $(1 - p_t)$ can be interpreted as a proxy for the probability of informed trading in the option market.

1.3.3 The third criterion: Hedging option position

Option trades for which the first two criteria show abnormal behavior cannot be immediately classified as informed trading. It could be the case that such transactions were

[3] Consider for example an option which exhibits an unusually high increment in open interest at time t, say $OI_{t-1} = 1000$ and $OI_t = 3000$, resulting in $X_t := OI_t - OI_{t-1} = 2000$. Suppose that in the days following this transaction the level of open interest decreases and after h days reaches the level $OI_{t+h} = 500$. One should only consider the gains realized through exercise till time $\tau_t \leq t + h$, where τ_t is such that the sum of negative decrements in open interest during $[t + 1, \tau_t]$ equals $X_t = 2000$.

hedged by traders using the underlying asset. Without knowing the exact composition of each trader's portfolio, it is not possible to assess directly whether each option trade was hedged or not. For example suppose that a trader buys a large number of stock, hedges this exposure buying put options, and the stock price indeed drops a few days later. Using the first two criteria, such a transaction in put option would be classified as informed. Another misclassification would occur in the opposite situation when the investor buys a large amount of put options and hedges her position by buying the suitable amount of the underlying stock.

We attempt to assess indirectly whether unusual trades in put options are actually delta hedged using the underlying asset. The idea is to compare the theoretical total amount of shares bought for non-hedging purposes and the total volume of buyer-initiated transactions in the underlying stock. If the latter is significantly larger than the former, then it is likely that some of the buyer-initiated trades occur for hedging purposes. In the opposite case we conclude that the new option positions are naked. The difficulty is that the volume due to hedging is typically a small component of the total buyer-initiated volume. To approximate this volume we assume that when hedging occurs and no informed trades take place, newly issued options are hedged on the same day. Moreover, a hedging analysis at the level of single option is not possible using the OptionMetrics database. We therefore check whether all the newly issued options are hedged on a specific day t. Given our definition of informed option trades, such trades certainly account for a large fraction of the newly issued options. For each day t, the total volume of the underlying stock is divided into seller- and buyer-initiated using intraday volumes and transaction prices according to the [30] algorithm.[4] Then the buyer-initiated volume, V_t^{buy}, is divided into volume due to hedging and to non-hedging purposes, $V_t^{\text{buy,hedge}}$ and $V_t^{\text{buy,non-hedge}}$, respectively. Let $\Delta_t^{P,k}$ be the delta of put option k and K_t^P the set of put option (newly issued or already existing) on day t. Similarly for $\Delta_t^{C,k}$ and K_t^C. Let

$$\alpha_t := \sum_{k \in K_t^P} |OI_t^{P,k} - OI_{t-1}^{P,k}| \cdot |\Delta_t^{P,k}| \, , \gamma_t := \sum_{k \in K_t^C} |OI_t^{C,k} - OI_{t-1}^{C,k}| \cdot \Delta_t^{C,k},$$

[4] The algorithm states that a trade with a transaction price above (below) the prevailing quote midpoint is classified as a buyer- (seller-) initiated trade. A trade at the quote midpoint is classified as seller-initiated if the midpoint moved down from the previous trade (down-tick), and buyer-initiated if the midpoint moved up (up-tick). If there was no movement from the previous price, the previous rule is successively applied to several lags to determine whether a trade was buyer- or seller-initiated.

$$\beta_t := \sum_{k \in K_t^P} |\Delta_t^{P,k} - \Delta_{t-1}^{P,k}| \cdot OI_{t-1}^{P,k}\,,\; \delta_t := \sum_{k \in K_t^C} |\Delta_t^{C,k} - \Delta_{t-1}^{C,k}| \cdot OI_{t-1}^{C,k}.$$

The α_t and γ_t represent the theoretical number of shares to buy for hedging the new options issued at time t, whereas β_t and δ_t are the theoretical number of shares to buy to rebalance the portfolio of existing options at time t. Absolute changes in open interests and deltas account for the fact that each option contract has a long and short side that follow opposite trading strategies if hedging occurs. The theoretical buyer-initiated volume of stock at time t for hedging purposes, $V_t^{\text{buy,hedge-theory}}$, is

$$V_t^{\text{buy,hedge-theory}} := \alpha_t + \beta_t + \gamma_t + \delta_t.$$

When the first two criteria of our method do not signal any informed trade, we approximate $V_t^{\text{buy,hedge}}$ by $V_t^{\text{buy,hedge-theory}}$. Then the amount of stock bought for non-hedging purposes is calculated as

$$V_t^{\text{buy,non-hedge}} = V_t^{\text{buy}} - V_t^{\text{buy,hedge-theory}}.$$

When informed option trades take place on day i, $V_i^{\text{buy,non-hedge}}$ cannot be computed as in the last equation because $V_i^{\text{buy,hedge-theory}}$ would be distorted by the option informed trades. We circumvent this issue by forecasting the volume $V_i^{\text{buy,non-hedge}}$ on day i using historical data on $V_t^{\text{buy,non-hedge}}$. The conditional distribution of $V_i^{\text{buy,non-hedge}}$ is estimated using the adjusted Nadaraja–Watson estimator and the bootstrap method proposed by [24]

$$\tilde{F}(y|\mathbf{x}) = \frac{\sum_{t=1}^{T} \mathbf{1}_{\{Y_t \le y\}} w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})}{\sum_{t=1}^{T} w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})} \tag{1.3}$$

with $Y_t := V_t^{\text{buy,non-hedge}}$, $\mathbf{X}_t := (|r_t|, V_{t-1}^{\text{buy,non-hedge}})$, $K_{\mathbf{H}}(.)$ being a multivariate kernel with bandwidth matrix $\mathbf{H}$, $w_t(\mathbf{x})$ the weighting function, and r_t the stock return at day t; we refer the reader to e.g. [22] for the implementation of (1.3).

We can now formally test the hypothesis, H_0, that hedging does not take place at day i. Whenever the observed V_i^{buy} is large enough, say above the 95% quantile of the predicted distribution of $V_i^{\text{buy,non-hedge}}$, it is likely that a fraction of V_i^{buy} is bought for hedging purposes. Hence we reject H_0 at day i when

$$V_i^{\text{buy}} > q_{0.95}^{V_i^{\text{buy,non-hedge}}}$$

where $q_\alpha^{V_i^{\text{buy,non-hedge}}} = \tilde{F}^{-1}(\alpha|\mathbf{X}_i)$ is the α-quantile of the predicted distribution of $V_i^{\text{buy,non-hedge}}$ estimated applying (1.3) to e.g. the last two years of data. Section 1.5.5 discusses the accuracy of the hedging detection method. We remark that the hypothesis H_0 of no hedging when informed trades occur refers to e.g. long positions in newly issued put options which are not hedged taking long positions in the underlying stock and motivating our hedging detection method. The corresponding short positions in the same put options might or might not be hedged, taking short positions in the underlying stock, without any impact on our hedging detection method. It is so because the total volume of the underlying stock is divided into buyer- and seller-initiated.

1.3.4 Detecting option informed trades combining the three criteria

Two methods are proposed to detect informed trades. The first method relies only on *ex-ante* information and is based on (C_1) changes in open interest and volume and (C_3) absence of hedging strategy using underlying asset. The second method uses information available before and after a given transaction, and is based also on (C_2) return and gain generated by the option trade. The first method aims at detecting informed trades as soon as they take place, while the second method allows for a more stringent assessment of informed trades. Let k_t denote the selected informed trade at day t in option k. The two methods can be succinctly described using the following sets of events

- *Ex-ante criteria C_1 and C_3:*

  $\Omega_1 := \{k_t \text{ such that } q_t \leq 5\%\}$

  $\Omega_2 := \{k_t \text{ such that } H_0 : \text{non-hedging, not rejected at day } t\}$

- *Ex-post criterion C_2:*

  $\Omega_3 := \{k_t \text{ such that } r_t^{\max} \geq q_{0.90}^{r_t^{\max}}\}$

  $\Omega_4 := \{k_t \text{ such that } G_t \geq q_{0.98}^{G_t}\}$.

The first method detects an informed option trade when it belongs to the first two sets, i.e. $k_t \in \Omega_1 \cap \Omega_2$, while according to the second method the selected informed trade belongs to all four sets, i.e. $k_t \in \Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4$. The empirical quantiles at day t of $r_t^{\max}$ and G_t distributions, $q_{0.90}^{r_t^{\max}}$ and $q_{0.98}^{G_t}$, are computed using the last two years of data.

As any other statistical method our detection methods could generate false discoveries, i.e. the probability that an option trade could satisfy the three criteria by chance is nonzero. It is well-known that this misclassification is not eliminable and corresponds to the Type I error in hypothesis testing. However our detection method is designed to be as conservative as possible minimizing the Type I error. As shown in Section 1.5, setting the input parameters properly only a handful of option trades are identified as informed, for e.g. less than 0.1%.

1.4 Data

Various databases are used in the empirical study. For KLM and thirteen American companies, options data are from the Chicago Board Options Exchange (CBOE) as provided by OptionMetrics. The dataset includes the daily cross section of available put options for each company from January 1996 to April 2006 and amounts to roughly 1.5 million of options. We eliminated obvious data errors such as open interest reported at zero for all existing options by excluding those days from our analysis. Stock prices are downloaded from OptionMetrics as well to avoid non-synchronicity issues and are adjusted for stock splits and spin-offs using information from the CRSP database. Intraday transaction prices and volumes for each underlying stock prices are provided by NYSE's Trade and Quote (TAQ) database. This database consists of several millions of records for each stock and is necessary to classify volumes in buyer- and seller-initiated. Discrepancies among datasets have been carefully taken into account when merging databases. For example data for J.P. Morgan from OptionMetrics and TAQ do not match. Whereas the stock volume reported in OptionMetrics for the years 1996–2000 is given by the sum of the volume of Chase Manhattan Corporation and J.P. Morgan & Co. (Chase Manhattan Corporation acquired J.P. Morgan & Co. in 2000), TAQ only reports the volume of J.P. Morgan & Co. Same issue was found for BankAmerica Corporation and NationsBank Corporation, whose merger took place in 1998 under the new name of Bank of America Corporation. Fourteen companies from airline, banking and various other sectors are analyzed. The list of companies includes: American Airlines (AMR), United Airlines (UAL), Delta Air Lines (DAL), Boeing (BA) and KLM for the airline sector; Bank of America (BAC), Citigroup (C), J.P. Morgan (JPM), Merrill Lynch (MER) and Morgan Stanley (MWD) for the banking sector; and AT&T (ATT), Coca-Cola (KO), Hewlett Packard (HP) and Philip Morris (MO) for the remaining sectors. Sample data range from January 1996 to April 2006. Options data for DAL and KLM were available only for somewhat shorter periods. For the analysis of European companies, Swiss Re, Munich RE and EADS, we use daily data from the EUREX provided by Deutsche Bank. Intraday data for such European companies were not available.

1.5 Empirical results

The proposed methods to detect option informed trades are applied to fourteen companies whose options are traded on the CBOE: AMR, UAL, DAL, BA and KLM (airline sector); BAC, C, JPM, MER and MWD (banking sector); and ATT, KO, HPQ and MO; see Section 2.3 for the ticker symbols. The first method which relies only on ex-ante information is already a powerful tool in order to detect potential informed trades as soon as they take place. On average, less than 0.1% of the total analyzed trades belongs to the set $\Omega_1 \cap \Omega_2$. For AMR, we found for example that the number of trades belonging to $\Omega_1 \cap \Omega_2$ is 141, the total number of analyzed options being more than 137,000. For the remaining companies, comparable numbers have been found. Due to space constrains we do not report the details of transactions belonging to $\Omega_1 \cap \Omega_2$ but these are available from the authors upon request. Based on the second method, the number of detected informed trades decreases substantially. Analyzing all daily cross sections of put options for all companies from January 1996 to April 2006, in total 37 transactions on the CBOE have been identified as belonging to the set $\Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4$; the total number of put option trades analyzed is roughly 1.5 million. Nearly all the events can be assigned to one of the following three event categories: merger and acquisition (M&A) announcements, 6 transactions; quarterly financial/earnings related statements, 14 transactions; and the terrorist attacks of September 11th, 13 transactions. 4 transactions could not be identified.

Table 1.1 summarizes the findings. 4 informed trades around M&A announcements are detected in the airline sector. These option trades have underlying stock American Airlines and United Airlines. Three informed trades took place on May 10th and 11th, 2000, two weeks before UAL's acquisition of US Airways was announced (for details see Footnote 1). Another informed trade took place on January 9th, 2003 with underlying Delta Air Lines, a few weeks before a public announcement on January 21st, 2003 related to the planned alliance among Delta, Northwest and Continental. In both cases, the underlying assets were strongly affected by the public announcements, generating large gains ($3 and $1 million, respectively) through the exercise of these put options.

Eight out of 15 of the selected transactions for the airline sector can be traced back to the terrorist attacks of 9/11. Companies like American Airlines, United Airlines, Boeing and to a lesser extent Delta Air Lines and KLM seem to have been targets for informed trading activities in the period leading up to the attacks. The number of new put options issued during that period is statistically high and the total gains G_t realized by exercising these options amount to more than $16 million. These findings support the results in [40] who also reports unusual activities in the option market before the terrorist attacks.

In the banking sector 14 informed trading activities are detected, 6 related to quarterly financial/earnings announcements, 5 to the terrorist attacks of September 11th, and 3 not identified. For example the number of new put options with underlying stock in Bank of America, Citigroup, J.P. Morgan and Merrill Lynch issued in the days before the terrorist attacks was at an unusually high level. The realized gains from such trading strategies are around $11 million.

The last set of companies we analyze includes AT&T, Coca Cola, Hewlett Packard and Philip Morris. Two informed trades occurred in the pre-announcement period of the M&A deal between Coca Cola and Procter&Gamble announced on February 21st, 2001 (leading to gains of more than $2 million), and 5 transactions preceding the publication of quarterly financial/earnings statements. Information related to earnings shortfalls, unexpected drops in sales and production scale backs are the most common in this last category. For example three informed trades in put options with underlying Philip Morris stock are detected. These trades took place a few days before three separate legal cases against the company seeking a total amount of more than $50 million in damages for smokers' deaths and inoperable lung cancer. The realized gains amounted to more than $10 million. Perhaps as expected, no informed option trade is detected with underlying the previous companies in the days leading up to the terrorist attacks of September 11th.

To provide a more detailed description of the detected informed trades, two tables are reported for every sector: Tables 1.2 and 1.3 for the airline sector; Tables 1.4 and 1.5 for the banking sector; and Tables 1.6 and 1.7 for the last group of companies. Tables 1.2, 1.4 and 1.6 report various information on the informed trades $k_t \in \Omega_1 \cap \Omega_3 \cap \Omega_4$, namely the day on which the transaction took place (Day); identification number of the put options (Id); the moneyness (S_t/K); its time-to-maturity (τ); the level of open interest the day before the informed transaction (OI_{t-1}); the increment in open interest from day $t-1$ to day t (ΔOI_t); its quantile with respect to its empirical distribution computed over the last two years ($q_t^{\Delta OI}$); the total increment in open interest (i.e. when considering all the available options at day t and not only the ones which had the highest increment, ΔOI_t^{tot}); the corresponding volume (Vol_t); the maximum return realized by the selected option in a two-week period following the transaction day ($r_t^{\max}$); the number of days between transaction day t and when this maximum return occurs (τ_2); the gains realized through the exercise of the new option issued at time t (G_t); the minimum between the number of days (starting from the transaction day) needed for the exercise of ΔOI_t and 30 days (τ_3); the percentage of ΔOI_t exercised within the first 30 days after the transaction ($\%ex.$); the ex-ante probability (q_t); the p-value of the hypothesis that hedging does not take place at time t; a proxy for the probability of informed trading ($1 - p_t$). Tables 1.3, 1.5 and 1.7 have a more descriptive nature and report the following information for the selected events: the day on which the transaction took place (Day of transaction); the market condition at day t given by the average return of the underlying stock during the last two trading weeks (Market condition); the minimum return of the underlying stock in a two-week period following the transaction day (Return); when the stock crashed (Crash in the stock); a short description of the event and why the stock dropped (Event's description). In most of the cases this drop in the underlying stock is large enough that its cause is reported in the financial press such as the business section of the New York Times. We could not identify the cause of a few events when the movements in the underlying stock were not significant. Interestingly, in most of these cases the hypothesis of non-hedging can be rejected at a 5% confidence level, suggesting that those option trades were not originated by informed traders. For transactions whose days are marked with asterisks the hypothesis of non-hedging can be rejected at a 5% level; see p-value reported on the last column of the corresponding tables.

Informed trades in the days leading up to quarterly financial statements might be somehow expected because the event day is known in advance. By definition, informed traders have either actively followed and analyzed the company's performance or are in possession of private information. Based on this knowledge they might therefore correctly guess the content of quarterly financial statements and develop profitable trading strategies. By contrast, the detected unusual activities in the options market before the terrorist attacks of September 11th and M&A public announcements deserve more attention. In what follows we concentrate therefore on these specific events. We analyze three cases in detail: the terrorist attacks of September 11th, the acquisition announcement in May, 2000 involving AMR and UAL, and the delay announcement of the EADS superjumbo jet A380. For the remaining selected trades one can do a similar analysis. To save space tables and figures are omitted but are available from the authors upon request.

1.5.1 The terrorist attacks of September 11th

The terrorist attacks have generated many articles, in which political, strategic and economic aspects have been considered. The financial dimension has also been discussed by the press. In particular, the question of whether the terrorist attacks of September 11th had been preceded by abnormal trading volumes, generated widespread news reports just after 9/11. As far as official regulators and control committees have been concerned, they dismiss charges against possible informed traders. The American 9/11 Commission has stated that “exhaustive investigations by the Security and Exchange Commission, FBI and other agencies have uncovered no evidence that anyone with advance knowledge of the attacks profited through securities transactions”.[5]

From an academic point of view, this topic did not generate much research interest. The article of [40] is a notable exception. Focused mainly on the airline sector, he computes the distributions of option market volume statistics both unconditionally and when conditioning on the overall level of option activity, the return and trading volume on the underlying stocks and the return on the overall market. He finds that “when the options market activity in the days leading up to the terrorist attacks is compared to the benchmark distributions, volume ratio statistics are seen to be at typical levels. As an indicator of long put volume, however, the volume ratio statistics appear to be unusually high which is consistent with informed investors having traded in the options market in advance of the attack”. In the following the informed option trades detected by our method are discussed in detail.

1.5.1.1 Analysis of options traded in CBOE

In total 13 transactions satisfy our criteria of informed trade and involve five airlines companies (AMR, UAL, BA and to a lesser extent DAL and KLM) and four banks (BAC, C, JPM and MER). Concerning the airline sector, AMR and UAL are the two companies

[5] The 9/11 Commission Report, Page 172, available on *http://www.9-11commission.gov/report/911Report.pdf.*

whose planes were hijacked and crashed by the terrorists. Informed option trade for KLM might be surprising, but supports the suspicion of "insider trading in KLM shares before September 11th attacks", as reported in a Dutch government investigation (Associated Press Worldstream). The terrorist attacks had indirect implications for BA and DAL, like a potential decrease in the number of passengers. Based on our methodology, AMR, UAL, and BA were more likely object of informed trade than DAL and KLM. With respect to the banking sector, Merrill Lynch, Bank of America, and J.P. Morgan were located in World Trade Center or nearby, and the Travelers Insurance Unit of Citigroup was expected to pay $500 million in claims.

In the case of American Airlines we will now report the details of the transaction which took place on September 10th. Additional tables are available from the authors upon request. The upper graphs in Figure 1.1 show the plot of option volume, V_t, versus its increment in open interest, X_t. The informed trades are highlighted with the circles. The left graph covers the period from January 1997 to December 2001, to better visualize the option market condition up to December 2001. The right graph covers the period January 1997–January 2006. The selected transactions are isolated from the bulk of the data, suggesting that they are statistically unusual. For September 2001 Figures 1.2 and 1.3 show the dynamic of three variables: open interest, volume and the option return. As claimed in several newspaper articles, the volume and open interest of puts had been unusually high in the days leading up to September 11th. On September 10th 1,535 put contracts were traded and from September 7th to September 10th the open interest increased of 1,312 contracts (at 99.5% quantile of its two-year empirical distribution, Figure 1.2). The trading volume was more than 60 times the average of the total daily traded volume during the three weeks before September 10th. These puts had a strike price of $30 and a maturity in October. On September 10th, the stock price was $29.7 and the put price was $2.15. On September 17th, when markets reopened after the attacks, the stock price was $18 and the put price was $12. Such an investment in put options generated an unusually high return (458% in one week). Put options were obviously exercised on September 17th, the open interest decreased of 597 contracts, generating a gain of almost $600,000. A few days later, another considerable number of put options (475 contracts) were exercised; see Figure 1.2. Table 1.2 reports the gains (G_t) of such a trade. Twenty-six days later the sum of exercised options corresponded to the increment observed on September 10th and lead to a cumulative gain of more than one million (G_t = $1,179,171). The lower graph in Figure 1.1 shows the cumulative gain for all transactions selected using the three criteria. The trade in put options of AMR corresponds to the transaction that leads to the highest gains in the shortest time interval in the period we are considering. Figure 1.2 shows that the trading volume after September 17th was negligible meaning that the main gain was realized through exercise and not selling the options. Similar conclusions can be reached for the other trades selected using our procedure. For example two trading days before the terrorist attacks 4,179 new put options (at 98.5% quantile of its two-year empirical distribution) on Boeing were issued. The underlying stock was traded at $45.18 and the option had a strike of $50. On September 17th, the stock was traded at $35.8. Six days afterwards these options were exercised leading to gains of more than five million. Concerning Bank of America, a large increment of 3,380 in open interest (at 96.3% quantile of its two-year empirical distribution) took place on September 7th for an option with a strike of $60 when the underlying asset had a value of $58.59 (on September 17th, the underlying stock had a value of $54.35). The exercise of those options in the following seven days resulted in net gains of almost two million; for Merrill Lynch, on September 10th, 5,615 new put options (at 99.1% quantile of its two-year empirical distribution) with strike $50 were issued, the underlying stock had a value of $46.85. On September 17th the underlying stock was traded at $41.48. Less than six days later these options had been exercised leading to gains of around $4.5 million. For the remaining companies similar results can be reached from the reported tables. Based on Tables 1.2 and 1.4, the total gains in the airline sector amount to more than $16 million, whereas in the banking sector $11 million in gains have been computed. Interestingly, in nearly all cases the hypothesis of non-hedging cannot be rejected.[6]

1.5.1.2 Analysis of options traded in EUREX

Several reinsurance companies suffered severe losses from the terrorist attacks of September 11th. Liabilities for Munich Re and Swiss Re—the world's two biggest reinsurers—were

[6] In the article "Not much stock in put conspiracy: the attacks on New York City and Washington have led to a new urban legend, namely that inside traders used put options on airline stocks to line terrorist pockets" published on June 3th, 2002 by Kelly Patricia O'Meara in *Insight on the News*, other repeated spikes of volumes of put options on American Airlines and United Airlines during the year before 9/11 are highlighted and used as argument that what occurred in the days leading up to 9/11 was not as unusual as other theories claim. Our method does not select any of those spikes mainly because of the relatively small gains that they generated.

estimated to be in the amount of billions of dollars a few days after the attacks. At the same time, several newspapers reported that trading in shares of these two companies were at unusual levels in the days leading up to September 11th, divulging some rumors of informed trading activities. A detailed analysis of transactions on the options market has however thus far been ignored. Options with underlying Swiss Re and Munich Re are mainly traded on the EUREX, one of the world's largest derivatives exchanges and the leading clearing house in Europe established in 1998 after the merger of Deutsche Terminbörse (DTB, the German derivatives exchange) and SOFFEX (Swiss Options and Financial Futures). In this section we use the EUREX database provided by Deutsche Bank to analyze transactions in put options with underlying Swiss Re and Munich Re. The database does not contain intraday data and hence the hedging dimension cannot be investigated.

In the case of Munich Re, 4 informed trades are detected between 1999 and 2008 which belong to the set $\Omega_1 \cap \Omega_3 \cap \Omega_4$, one of which took place on August 30th, 2001. As we are mainly interested in informed trades surrounding the terrorist attacks in this subsection, we only discuss the details of this transaction (the others took place on August 29th, 2002; September 2nd, 2002; and October 19th, 2007). The detected put option with underlying Munich Re matured at the end of September, 2001 and had a strike of €320 (the underlying asset was traded at €300.86 on August 30th). That option shows a large increment in open interest of 996 contracts (at 92.2% quantile of its two-year empirical distribution) on August 30th. Its price on that day was €10.22 and the ex-ante probability q_t is slightly lower than 5%. On the day of the terrorist attacks, the underlying stock lost more than 15% (the closing price on September 10th was €261.88 and on September 11th €220.53) and the option price jumped to €89.56, corresponding to a return of 776% in 8 trading days. On September 12th, 1,350 put options with those characteristics were exercised. The gains G_t related to the exercise of the 996 new put options issued on August 30th correspond to more than €3.4 million.

In the case of Swiss Re, 6 informed trades are detected between 1999 and 2008 which belong to the set $\Omega_1 \cap \Omega_3 \cap \Omega_4$, one of which took place a few weeks before the terrorist attacks, on August 20th. This option expired at the end of September, 2001, had a strike of €159.70 and had a large increment in open interest of 3,302 contracts (at 99.8% quantile of its two-year empirical distribution) on August 20th. That option was traded at €0.8 and exhibits an ex-ante probability q_t of 0.4%, meaning that such an event happens on average once every year. The Swiss Re closing share price was €177.56 on August 20th. On September 11th, when the stock price fell from €152.62 to €126.18, the option generated a return of 4,050% in three trading weeks, when its price jumped to €33.2. Through the exercise of these new put options in the 9 days following the attacks, the total gains were more than €8 million. Together with Munich Re, a total gain of €11.4 million had been realized in less than two trading weeks by using two options with underlying Munich Re and Swiss Re. To save space the corresponding tables and figures are omitted but are available from the authors upon request.

1.5.2 The acquisition announcement in the US airline sector in May 2000

Two informed trades detected by our method took place on May 10th and 11th, 2000. They involved AMR and UAL. On May 10th and 11th, the number of new options issued with strike \$35 and maturity June 2000 with underlying AMR is very large: 3,374 on May 10th and 5,720 the day after (at 99.7% and 99.9% quantile of their two-year empirical distributions, respectively). These transactions correspond to those which exhibit the strongest increments in open interest during a span of five years; see upper left graph in Figure 1.1 and Figure 1.3. On May 10th, the underlying stock had a value of \$35.50 and the selected put was traded at \$2.25. For UAL 2,505 new put options (at 98.7% quantile of its two-year empirical distribution) with strike \$65 and the same maturity as those of AMR were issued on May 11th at the price of \$5.25 when the underlying had a value of \$61.50. The market conditions under which such transactions took place do not show any particularity: the average return of the stock the week before is, in both cases, positive and less than 0.5%. The days of the drop in the underlying stock are May 24th and May 25th, 2000, with the first day corresponding to the public announcement of United Airline's regarding a \$4.3 billion acquisition of US Airways. As reported in the May 25th, 2000 edition of the New York Times, "shares of UAL and those of its main rivals crashed" (for details see Footnote 1). The stock price of AMR dropped to \$27.13 (−23.59% of value losses when compared to the stock price on May 11th) increasing the value of the put options to \$7.88 (resulting in a return of 250% in two trading weeks). The same impact can be found for UAL: the stock price after the public announcement dropped to \$52.50 (−14.63% when compared to the value on May 11th) raising the put's value to \$12.63 (corresponding to a return of 140% in two trading weeks). In the case of AMR, the decline in the underlying stock can be seen in Figure 1.3, where the option return largely increased. On the day of the public announcement 4,735 put options of AMR were exercised; see Figure 1.3. After this large decrement in open interest, 1,494 and 1,376 additional put options were exercised in the following two days respectively (reflected in additional drops in open interests in Figure 1.3). The unusual increments in open interest observed on May 10th and May 11th are therefore off set by the exercise of options when the underlying crashed. The corresponding gains G_t from this strategy are more than \$1.6 million within two trading weeks. These are graphically shown in the lower graph in Figure 1.1, from which we can see how fast these gains were realized. In the case of UAL similar conclusions can be reached; see Tables 1.2 and 1.3. Based on these trades, a total gain of almost \$3 million was realized within a few trading weeks using options with underlying AMR and UAL. The non-hedging hypothesis cannot be rejected suggesting that such trades are naked option positions.

1.5.3 The delayed delivery announcement of EADS superjumbo A380 in May 2006

At the time of the writing of this paper, European Aeronautic Defence and Space (EADS), a large European aerospace corporation and the parent of plane maker Airbus, is under investigation for illegal insider trading activities. On July 2nd, 2006, co-CEO Noël Forgeard and Airbus CEO Gustav Humbert resigned following the controversy caused by the June 14th, 2006 announcement that deliveries of the superjumbo jet A380 would be delayed by a further six months. Mr. Forgeard was one of a number of executives who sold his stake in EADS a few months before the public announcement. In June shares of EADS exhibited a 26% fall (the closing price of EADS shares on June 13th was €25.42 and on June 14th €18.73) wiping more than €5 billion from the company's market value. He and 21 other executives are currently under investigation as to whether they knew about the delays in the Airbus A380 project and sold their stock on the basis of this private information, constituting therefore illegal insider trading. In the financial press, the profits resulting from this strategy are estimated to total approximately €20 million.[7]

Based on reports in the financial press, French authorities' investigations have concentrated thus far on stock sales and stock options exercised before the announcement day. Apparently, trading strategies based on put options were ignored, despite their appealing features for investors in possession of private information. We apply our method to put

[7] The New York Times edition of June 18th, 2008: "Executive Questioned in EADS Insider Trading Case".

options on EADS and detect various informed trades on the EUREX in the period leading up to the announcement day. Obviously our study does not constitute proof of illegal activities.

For the period 2003–2009 our procedure detects six informed trades in put options belonging to the set $\Omega_1 \cap \Omega_3 \cap \Omega_4$, all of which took place between April 6th and May 19th, 2006.[8] Table 1.8 summarizes the findings. Four of these six options had maturity at the end of June 2006, the remaining two end of May 2006 and end of July 2006. The four options maturing in June 2006 exhibited large increments in open interest and volume on April 7th (3,855 contracts), on April 20th (1,000 contracts), on May 8th (810 contracts) and on May 18th (2,518 contracts). These increments correspond to the 99.8%, 93.4%, 92.2%, and 99% quantiles of the corresponding two-year empirical distributions. The options had strikes of €32, €30, €30 and €31 and the underlying traded at €31.88, €31.30, €31.36 and €27.59 respectively on the transaction days. The maximum returns generated from these trades are large: for example, the option selected on May 8th traded at €0.71 on that day and on June 14th its price jumped to €11.27 when the stock crashed. This corresponds to a return of 1,487% within five trading weeks. On the announcement day 760 contracts of that option were exercised, generating a net profit of €802,560. The option selected on May 18th, traded at €3.46 on that day and at €12.27 on June 14th, resulting in a return of 255% within four trading weeks. On June 16th 2,667 contracts were exercised. Assuming that the 2,518 options issued on May 18th were exercised on that day, a net gain of €1.7 million is reached. The option with a large increment in open interest on May 19th and maturity end of July was bought for €0.71 on that day and had a strike of €26 when the underlying traded at €27.39. On the announcement day its value increased to €7.27, corresponding to a net return of 924% within four trading weeks. After the announcement day, these options were exercised and generated a net gain of almost €1.5 million. Similar patterns are observed for the options traded on April 7th (strike €32 and underlying value €31.88): the large increment of 3,855 contracts generated total gains of almost €1.7 million. For the remaining options, a similar analysis can be made. Figure 1.5 shows the corresponding realized gains. Figures 1.6 and 1.7 show relevant variables for the

[8] On May 12th, 2006, a meeting of the company board took place in Amsterdam in order to discuss possible solutions to the management crisis triggered by the future announcement day. This was planned to take place the following month. According to the New York Times edition of June 29th, 2006, 13 people were present, including Noël Forgeard and Gustav Humbert. The delay in A380 deliveries was likely to cost EADS €2 billion over the following four years.

transactions which took place on April 20th and May 19th, 2006; see also Table 1.8. Based on the six detected transactions, a total gain of €7.5 million had been realized within 60 trading days after the announcement.[9]

1.5.4 Robustness checks

The input parameters in our detection procedure are: the length N of the estimation window, chosen to be $N = 500$ trading days, used for the computation of the ex-ante probability q_t, the conditional distribution of $V_t^{\text{buy,non-hedge}}$, and the quantiles $q_\alpha^{r_t^{\max}}$ and $q_{\alpha'}^{G_t}$; the time period after the transaction day used for the computation of R_t, chosen to be 10 trading days; the time horizon τ_t used for the calculation of the gains G_t, chosen to be 30 trading days; the quantile levels α and α' in $q_\alpha^{r_t^{\max}}$ and $q_{\alpha'}^{G_t}$ used for the computation of the sets Ω_3 and Ω_4, chosen to be $\alpha = 90\%$ and $\alpha' = 98\%$; the probability level based on which we select trades belonging to the set Ω_1, chosen to be 5% in our selection procedure. In what follows we set the input parameters to different values and we repeat all previous analysis for all companies. To save space we report only some of the results but the remaining ones are available from the authors upon request.

When varying the length of the estimation window N between 200 and 1,000, (all other parameters being unchanged) the number of selected transactions does not change significantly. For example in the case of AMR, we selected 5 informed trades when considering the last two trading years ($N = 500$ days); for $N \in [200, 1000]$ the number of detected informed trades ranges between 4 and 6; for UAL this number remains unchanged with respect to the original choice for $N > 450$ and decreases by one when $N \in [200, 450]$. In the case of BAC and AT&T, the deviation from the original number of selected trades is less than 2. With respect to the choice of the time period used for the computation of R_t and τ_t, our results are also robust. We let the length of the first period vary in the range $[1, 30]$ days and the second one in [1,40] days. In the case of AMR, the number of transactions ranges from 2 to 8, being therefore centered around the original number and with a small deviation from it. For UAL, the corresponding range is from 1 to 4, for BAC from 2 to 8 and for AT&T from 1 to 6. The number of detected trades is obviously a

[9] Options contracts with underlying EADS are traded at the EURONEXT in Paris as well. Using a database provided by EURONEXT NYSE, we were able to apply the first two criteria of our detection procedure. Six informed put options trades were identified in Spring 2006. The total gains collected amount at €25.6 million.

decreasing function of α and α' (all other parameters being unchanged). In the case of AMR, when $\{\alpha, \alpha'\} \in [0.85, 0.95] \times [0.96, 1]$, the number of transactions selected does not exceed 15. For UAL, the number of selected trades varies between 1 and 10, for BAC between 5 and 25, and for AT&T between 1 and 18. Finally, with respect to the probability level used to determine the set Ω_1, our findings are very robust as well. When increasing the level from 1% to 10%, the number of trades selected for AMR varies between 1 and 6; for UAL it ranges between 2 to 4, for BAC and AT&T from 1 to 7. We simultaneously changed several parameters and found that the number of detected transactions does not change significantly and in almost all cases in steps of one. We recall that approximately 1.5 million of options are analyzed. Based on these results, we conclude that our findings are robust.

1.5.5 Accuracy of the hedging detection method

In this section we provide an assessment of the accuracy of our hedging detection method introduced in Section 2.2.3. Recall that the hypothesis H_0 of no hedging when informed trades occur at day i is rejected whenever $V_i^{\text{buy}} > q_\alpha^{V_i^{\text{buy,non-hedge}}}$, suggesting that a sizable component of buyer-initiated trades in the stock is due to hedging. We measure the accuracy of the method by computing the probability of rejecting H_0 when the latter does not hold, i.e. the power of the test. Let $V_i^{\text{buy}} = (1 + h_i)\ V_i^{\text{buy,non-hedge}}$, where $h_i \geq 0$. The h_i represents the ratio between buyer-initiated volume due to hedging and buyer-initiated volume due to non-hedging. By construction H_0 is equivalent to $h_i = 0$ meaning that volume trades due to hedging is zero. The hypothesis H_0 should be rejected when $h_i > 0$, and the higher the rejection rate the more accurate the hedging detection method. Let $q_\alpha := q_\alpha^{V_i^{\text{buy,non-hedge}}}$, the measure of accuracy $\mathbb{A}(h_i)$ reads therefore

$$\mathbb{A}(h_i) := \mathbb{P}\Big[V_i^{\text{buy}} > q_\alpha | h_i\Big] = \mathbb{P}\Big[V_i^{\text{buy,non-hedge}} > q_\alpha/(1 + h_i) | h_i\Big]. \tag{1.4}$$

The hedging detection method is accurate whenever $\mathbb{A}(h_i)$ increases fast enough in h_i. The probability in (1.4) can be calculated as $(1 - \tilde{F}(q_\alpha/(1+h_i)|\mathbf{X}_i))$, where $\tilde{F}$ is estimated using (1.3) and $\alpha = 0.95$ as in our empirical analysis. We computed $\mathbb{A}(h_i)$ for several stocks, sample periods, estimation windows, and different values of h_i and of the conditioning variables $\mathbf{X}_i = (|r_i|, V_{i-1}^{\text{buy,non-hedge}})$. Table 1.9 gives numerical values of $\mathbb{A}(h_i)$ for Citigroup on the random day December 17th, 2001. Corresponding results for other stocks are fairly similar and available upon request from the authors. When $h_i = 0$, $\mathbb{A}(h_i)$ is very close to $0.05 = (1 - \alpha)$, which is the non-eliminable size of the test. When h_i increases, $\mathbb{A}(h_i)$ increases as well although certain combinations of the conditioning variables are more favorable than others to reject the hypothesis of no hedging. Overall the power of the test is fairly satisfactory. For example when $h_i = 0.20$, $\mathbb{A}(h_i)$ can be as high as 20%. When $\mathbb{A}(h_i)$ does not increase fast enough, our method does not detect potential option informed trades. In this respect the results documented in the empirical section should be interpreted in a conservative manner.

1.6 Conclusion

Informed trading activities in stock markets have been extensively investigated in the finance literature. Our paper contributes to this literature in two directions: it studies informed trading activities in option rather than stock markets and provides a statistical method to detect informed trades in option contracts. According to our method, an option trade is identified as informed when it is characterized by a large increment in open interest and volume, induces large gains, and is not hedged in the stock market. This method is applied to each put option contract on 14 companies in various business sectors traded in the Chicago Board Options Exchange from January 1996 to April 2006 analyzing approximately 1.5 million of options. In total 37 transactions are identified as informed trades the vast majority of which can be assigned to one of the following three event categories: merger and acquisition announcements, quarterly financial/earnings related statements, and the terrorist attacks of September 11th. For example two informed trades involve American Airlines and United Airlines on May 10th and 11th, 2000, namely two weeks before UAL's acquisition of US Airways was announced. Three informed trades on put options with underlying Philip Morris stock are detected a few days before three separate legal cases against the company seeking a total amount of more than \$50 million in damages for smokers' deaths and inoperable lung cancer. Our method is also applied to each put option on Swiss Re, Munich Re and EADS traded on EUREX from January 1999 to January 2008. For example in the case of EADS, the parent of plane maker Airbus, six informed option trades are identified between April and June 2006. These trades precede the June 14th, 2006 announcement that deliveries of the superjumbo jet A380 would be delayed by a further six months, causing a 26% fall in the underlying stock, and a total gain of €7.5 million in these option trades.

Our results have also policy, option pricing, and market efficiency implications. If some of the detected informed trades are indeed illegal, for example originated by insiders, it might be optimal for regulators to expend relatively more monitoring efforts on the options markets. Option pricing models should account for all relevant information available at time t. However nearly all option prices involved in informed trades according to our method do not show any specific reaction to the large increments in open interest and volume. The strong increases in these put option prices are simply due to subsequent large drops in stock prices originated for example by merger and acquisition announcements. From an efficient market perspective, our findings suggest that certain put option trades might predict large price drops. Trading strategies built on such predictions might generate potentially large gains.

Summary of Airline, Banking and Various sectors Jan 1996 - Apr 2006

	Airline		Banking		Various		Total
merger and acquisition announcement	4 (4)	22.22% (26.67%)	0 (0)	0.00% (0.00%)	2 (2)	8.00% (25.00%)	**6 (6)**
quarterly financial/earning related announc.	3 (3)	16.67% (20.00%)	15 (6)	55.56% (42.86%)	18 (5)	72.0% (62.50%)	**36 (14)**
terrorist attacks of September 11	10 (8)	55.56% (53.33%)	5 (5)	18.52% (35.71%)	0 (0)	0.00% (0.00%)	**15 (13)**
not identified	1 (0)	5.56% (0.00%)	7 (3)	25.93% (21.43%)	5 (1)	20.00% (12.50%)	**13 (4)**
Total	**18 (15)**		**27 (14)**		**25 (8)**		**70 (37)**

Table 1.1. Number of transactions identified as informed, percentage for the various sectors and corresponding event category. An informed put option trade is characterized by a statistically high increment in open interest and volume, generates an abnormal return and large gain a few days later and is not delta hedged. Entries refer to informed trades when disregarding the hedging dimension and when considering it (number in brackets when hedging demand is taken into consideration).

Summary of Airline Sector Jan 1996 - Apr 2006

Day	*Id*	$	τ	OI_{t-1}	ΔOI_t	$q_t^{\Delta OI}$	ΔOI_t^{tot}	Vol$_t$	$r_t^{\max}$	τ_2	G_t	τ_3	$\%ex.$	q_t	p-value	$1-p_t$
American Airlines (AMR) Jan 1996 - Apr 2006																
10 May 00	10821216	1.01	38	20	3374	99.7%	3378	3290	106%	9	906,763	11	100%	0.002	0.286	0.998
11 May 00	10821216	1.02	37	3394	5720	99.9%	5442	5320	98%	10	1,647,844	11	100%	0.002	0.349	0.998
31 Aug 01	20399554	0.91	22	96	473	95.7%	571	500	455%	7	662,200	11	100%	0.016	0.645	0.984
10 Sep 01	20428354	0.99	40	258	1312	98.5%	1701	1535	453%	2	1,179,171	26	100%	0.012	0.096	0.998
24 Aug 05	27240699	0.97	24	1338	4378	93.5%	8395	5319	163%	8	575,105	17	100%	0.048	0.123	0.952
United Airlines (UAL) Jan 1996 - Jan 2003																
11 May 00	11332850	0.95	37	35	2505	98.7%	2534	2505	132%	10	1,156,313	26	100%	0.002	0.373	0.998
6 Sep 01	20444473	1.06	44	21	1494	96.3%	1189	2000	1322%	7	1,980,387	28	100%	0.030	0.165	0.998
Delta Air Lines (DAL) Jan 1996 - May 2005																
*1 Oct 98	10904865	1.01	16	140	974	97.7%	483	924	261%	6	537,594	12	100%	0.016	0.000	0.996
29 Aug 01	20402792	0.98	24	1061	202	89.7%	224	215	1033%	9	328,200	13	100%	0.044	0.528	0.998
19 Sep 02	20718332	0.99	30	275	1728	98.7%	550	1867	132%	7	331,676	22	100%	0.004	0.190	0.998
9 Jan 03	21350972	1.10	44	274	3933	99.7%	4347	4512	112%	9	1,054,217	30	100%	0.002	0.065	0.998
Boeing (BA) Jan 1996 - Apr 2006																
24 Nov 98	10948064	0.99	53	3758	1047	93.5%	1285	1535	467%	7	883,413	24	100%	0.040	0.481	0.996
29 Aug 01	20400312	0.92	24	1019	2828	96.7%	3523	3805	382%	10	1,972,534	8	100%	0.028	0.252	0.998
5 Sep 01	20429078	1.01	45	472	1499	92.1%	2538	1861	890%	8	1,805,929	22	100%	0.048	0.085	0.998
6 Sep 01	11839316	0.75	135	13228	7105	99.3%	13817	7108	118%	7	2,704,701	3	100%	0.006	0.150	0.998
*7 Sep 01	20400311	0.90	15	7995	4179	98.5%	4887	5675	306%	6	5,775,710	7	100%	0.016	0.000	0.998
*17 Sep 01	20400309	0.90	5	116	5026	98.9%	2704	5412	124%	4	2,663,780	5	100%	0.010	0.000	0.998
KLM Jan 1996 - Nov 2001																
5 Sep 01	20296159	0.91	17	3	100	99.3%	34	100	467%	9	53976	9	100%	0.006	0.368	0.998

Table 1.2. Description of detected informed trades for the airline sector. For definition of entries see Page 38.

Summary of Airline Sector Jan 1996 - Apr 2006

Day of transaction	Market condition	Return	Crash in stock	Event's description
		American Airlines (AMR) Jan 1996 - Apr 2006		
10 May 00	0.4%	−17.6%	24/25 May 00	Announcement 24 May 00: Airline Deal UAL's acquisition of US Airways
11 May 00	0.0%	−17.6%	24/25 May 00	Announcement 24 May 00: Airline Deal UAL's acquisition of US Airways
31 Aug 01	−0.4%	−39.4%	17 Sep 01	9/11 Terrorist attacks in New York
10 Sep 01	−1.4%	−39.4%	17 Sep 01	9/11 Terrorist attacks in New York
24 Aug 05	0.4%	−5.3%	30 Aug 05	August 05: Hurricane Katrina, interrupted production on the gulf coast, jet fuel prices ↑
		United Airlines (UAL) Jan 1996 - Jan 2003		
11 May 00	0.3%	−12%	24 May 00	Announcement 24 May 00: Airline Deal UAL's acquisition of US Airways
6 Sep 01	−1.0%	−43.2%	17 Sep 01	9/11 Terrorist attacks in New York
		Delta Air Lines (DAL) Jan 1996 - May 2005		
*1 Oct 98	−1.7%	−11.4%	07/08 Oct 98	Not identified
29 Aug 01	0.0%	−44.6%	17 Sep 01	9/11 Terrorist attacks in New York
19 Sep 02	−5.2%	−24.4%	27 Sep 02	Announcement 27 Sep 02: Expected loss for 3rd quarter
9 Jan 03	2.1%	−15.7%	21/22 Jan 03	Announcement 21 Jan 03: Restrictions on planned alliance of Delta, Northwest and Continental
		Boeing (BA) Jan 1996 - Apr 2006		
24 Nov 98	−0.2%	−22.0%	02/03 Dec 98	Announcement 02. Dec 98: production scale back and cut in work forces
29 Aug 01	−0.4%	−25.0%	17/18 Sep 01	9/11 Terrorist attacks in New York
5 Sep 01	−0.8%	−25.0%	17/18 Sep 01	9/11 Terrorist attacks in New York
6 Sep 01	−0.9%	−25.0%	17/18 Sep 01	9/11 Terrorist attacks in New York
*7 Sep 01	−1.9%	−25.0%	17/18 Sep 01	9/11 Terrorist attacks in New York
*17 Sep 01	−5.6%	−25.0%	17/18 Sep 01	9/11 Terrorist attacks in New York
		KLM Jan 1996 - Nov 2001		
5 Sep 01	−1.9%	−31.6%	17/18 Sep 01	9/11 Terrorist attacks in New York

Table 1.3. Summary of detected informed trades for the airline sector. For definition of entries see Page 38.

Content of Tables 1.2, 1.4 and 1.6: day on which the transaction took place (Day); identification number (Id) of the put options; moneyness ($= S_t/K$); its time-to-maturity (τ); level of open interest the day before the informed trade (OI_{t-1}); increment in open interest from day $t-1$ to day t (ΔOI_t); its quantile with respect to its empirical distribution computed over the last two years ($q_t^{\Delta OI}$); total increment in open interest (i.e. when considering all the available options at day t and not only the ones which had the highest increment, ΔOI_t^{tot}); corresponding volume (Vol_t); maximum return realized by the selected option during the two-week period following the transaction day ($r_t^{\max}$); number of days between transaction day t and when this maximum return occurs (τ_2); gains realized through the exercise of the new option issued at time t (G_t); minimum between the number of days (starting from the transaction day) needed for the exercise of ΔOI_t and 30 days (τ_3); percentage of ΔOI_t exercised within the first 30 days after the transaction; ex-ante probability (q_t); p-value of the hypothesis that delta hedging does not take place at time t; proxy for the probability of informed trading ($1 - p_t$).

Content of Tables 1.3, 1.5 and 1.7: day on which the transaction took place (Day); market condition at day t measured by the average return of the underlying stock during the last two trading weeks (Market condition); minimum return of the underlying stock during the two-week period following the transaction day (Return, comparable therefore with $r_t^{\max}$ of the previous tables); short description of the event and why the stock drops (Event's description). In most of the cases this drop in the underlying stock is large enough that its cause is reported in the financial press such as the business section of the New York Times. The cause of a few informed trades could not be identified. In those cases the movements in the underlying stock were not significant and in several of these cases the hypothesis of non-hedging can be rejected at a 5% confidence level. For transactions whose days are marked with asterisks the hypothesis of non-hedging can be rejected at a 5% level; see p-value reported in the last column of the corresponding tables.

Summary of Banking Sector Jan 1996 - Apr 2006

Day	*Id*	\$	τ	OI_{t-1}	ΔOI_t	$q_t^{\Delta OI}$	$\Delta OI_t^{\mathrm{tot}}$	Vol_t	$r_t^{\max}$	τ_2	G_t	τ_3	*%ex.*	q_t	p-value	$1-p_t$
Bank of America (BAC) Jan 1996 - Apr 2006																
13 Jun 00	10196393	0.93	39	272	1996	94.10%	1883	2124	154%	7	1,505,256	28	100%	0.026	0.170	0.998
*13 Nov 00	11596097	1.00	5	1747	6273	99.10%	6240	7270	522%	5	3,081,216	5	100%	0.006	0.047	0.998
7 Sep 01	20400334	0.98	15	8720	3380	96.30%	3607	4303	241%	7	1,774,525	7	100%	0.026	0.091	0.994
Citigroup (C) Jan 1996 - Apr 2006																
30 Aug 01	20201221	1.07	23	9394	4373	94.50%	8880	5427	622%	10	2,045,940	12	100%	0.044	0.096	0.998
*18 Jun 02	20576902	0.96	95	3552	9984	97.90%	−8249	10090	114%	7	7,661,724	30	65%	0.002	0.000	0.998
*17 Jul 02	20732009	0.92	31	4467	4923	91.30%	9420	5148	227%	5	3,579,435	5	100%	0.028	0.000	0.996
28 Apr 04	21436285	0.97	24	38184	17803	99.90%	24618	21429	102%	9	3,172,024	18	100%	0.002	0.197	0.998
J.P. Morgan (JPM) Jan 1996 - Apr 2006																
*5 Oct 00	11674068	0.99	16	4632	2957	94.70%	3587	2843	391%	10	1,411,934	12	100%	0.030	0.004	0.998
*9 Nov 00	11848514	0.98	37	9303	9564	99.30%	10949	10681	164%	10	1,937,044	12	100%	0.004	0.000	0.998
29 May 01	11848586	0.99	18	22044	4290	95.70%	6603	5569	204%	9	1,508,490	10	100%	0.026	0.060	0.996
30 Aug 01	20435891	0.98	51	1370	3145	90.90%	2854	3407	153%	10	1,318,638	30	99%	0.026	0.058	0.998
6 Sep 01	20207536	0.92	16	22459	4778	96.30%	−9130	5359	178%	8	1,415,825	8	100%	0.014	0.075	0.998
18 Jan 02	20556357	1.03	29	6543	6168	97.10%	−85172	8421	225%	7	2,007,110	20	100%	0.024	0.145	0.996
17 Jan 03	21343021	0.95	36	5159	9597	99.10%	−133082	10527	117%	9	2,414,176	24	100%	0.006	0.061	0.998
Merrill Lynch (MER) Jan 1996 - Apr 2006																
*21 Aug 98	10840556	1.05	29	211	3679	99.50%	−6048	4165	428%	10	5,318,200	20	100%	0.002	0.000	0.998
*25 Aug 98	10963647	1.02	25	1410	1962	95.90%	2486	2207	629%	9	2,378,481	14	100%	0.020	0.000	0.998
*28 Aug 98	10840556	0.92	22	5138	2951	98.70%	2735	4703	186%	9	2,143,600	15	100%	0.012	0.000	0.996
*1 Sep 98	11499596	0.96	18	349	2224	96.70%	−1534	2548	136%	7	1,567,550	8	100%	0.014	0.000	0.998
10 Sep 01	20408663	0.94	12	6210	5615	99.10%	9898	7232	243%	5	4,407,171	6	100%	0.008	0.080	0.998
9 Apr 02	20642300	1.04	39	2549	3118	94.50%	5545	3513	129%	3	1,591,786	20	100%	0.010	0.135	0.998
Morgan Stanley (MWD) Jan 1996 - Apr 2006																
17 Aug 98	10174742	1.02	33	1003	1650	99.50%	1779	1660	341%	10	2,050,938	15	100%	0.004	0.197	0.998
*21 Aug 98	10148491	1.03	29	293	2064	99.70%	−3616	2362	554%	10	1,906,663	20	100%	0.004	0.005	0.998
25 Aug 98	10174742	0.98	25	2586	1291	98.70%	1638	2170	674%	9	1,467,850	6	100%	0.014	0.173	0.998
*28 Aug 98	11599638	0.93	22	2010	2010	99.50%	862	2010	265%	9	1,580,556	15	100%	0.002	0.000	0.998
3 Nov 00	10297869	1.04	15	4154	2297	97.90%	3285	3518	437%	8	1,947,447	11	100%	0.020	0.161	0.998
*22 May 01	20310213	1.06	25	1098	1816	90.30%	2284	1929	472%	6	1,871,086	18	100%	0.024	0.041	0.998
*6 Apr 05	31518375	1.03	10	14497	13807	99.90%	18342	18163	576%	8	2,780,148	8	100%	0.002	0.026	0.998

Table 1.4. Description of detected informed trades for the banking sector. For definition of entries see Page 38.

Summary of Banking Sector Jan 1996 - Apr 2006

Day of transaction	Market condition	Return	Crash in stock	Event's Description
			Bank of America (BAC) Jan 1996 - Apr 2006	
13 Jun 00	−1.0%	−14.8%	15/16 Jun 00	Announcement 15 Jun 00: Wachovia Corp. Correction of expected earnings for 2nd quarter
*13 Nov 00	−0.4%	−11.7%	14/15 Nov 00	Announcement 14 Nov 00: 3rd quarterly financial statements, potential write-offs for 4th quarter
7 Sep 01	−0.4%	−5.7%	17 Sep 01	9/11 Terrorist attacks in New York
			Citigroup (C) Jan 1996 - Apr 2006	
30 Aug 01	−0.5%	−6.7%	17 Sep 01	9/11 Terrorist attacks in New York
*18 Jun 02	0.6%	−5.4%	26 Jun 02	Not identified
*17 Jul 02	−0.3%	−26.7%	22/23 Jul 02	Announcement 22 Jul 02: Senate's investigations into Citigroup (Enron case)
28 Apr 04	−0.3%	−2.8%	10 May 04	Not identified
			J.P. Morgan (JPM) Jan 1996 - Apr 2006	
*5 Oct 00	−0.3%	−7.0%	12 Oct 00	Not identified
*9 Nov 00	−0.6%	−4.2%	15 Nov 00	Not identified
29 May 01	0.4%	−3.4%	6 Jun 01	Not identified
30 Aug 01	−0.8%	−7.5%	20 Sep 01	9/11 Terrorist attacks in New York
6 Sep 01	−1.5%	−7.5%	20 Sep 01	9/11 Terrorist attacks in New York
18 Jan 02	−1.4%	−6.6%	29 Jan 02	Announcement 16/22 Jan 02: financial statements for 4th quarter/losses on Enron's loans
17 Jan 03	−0.7%	−5.3%	24 Jan 03	Announcement 22 Jan 03: bigger 4th quarter loss than forecasted
			Merrill Lynch (MER) Jan 1996 - Apr 2006	
*21 Aug 98	0.0%	−16.3%	28/30/31 Aug 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
*25 Aug 98	−0.4%	−16.6%	09/10 Sep 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
*28 Aug 98	−2.6%	−16.6%	09/10 Sep 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
*1 Sep 98	−3.7%	−16.6%	09/10 Sep 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
10 Sep 01	−1.2%	−15.5%	17/18 Sep 01	9/11 Terrorist attacks in New York
9 Apr 02	−0.9%	−7.9%	11 Apr 02	Announcement 09 Apr 02: accusations of conflicts of interest, potential fine of > \$100mio
			Morgan Stanley (MWD) Jan 1996 - Apr 2006	
17 Aug 98	0.7%	−17.2%	28/31 Aug 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
*21 Aug 98	−0.3%	−17.2%	28/31 Aug 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
25 Aug 98	−0.5%	−17.2%	28/31 Aug 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
*28 Aug 98	−3.3%	−17.2%	28/31 Aug 98	Announcement 17 August 98: Ruble crisis, Russian crisis, Asian crisis
3 Nov 00	1.3%	−12.2%	07/08/09 Nov 00	Not identified
*22 May 01	2.3%	−5.7%	30 May 01	Not identified
*6 Apr 05	1.0%	−3.0%	20 Apr 05	Announcement 05 Apr 05: proposal of new CEO, discover credit card unit spin off

Table 1.5. Summary of detected informed trades for the banking sector. For definition of entries see Page 38.

Summary of various sectors Jan 1996 - Apr 2006

Day	*Id*	\$	τ	OI_{t-1}	ΔOI_t	$q_t^{\Delta OI}$	$\Delta OI_t^{\mathrm{tot}}$	Vol_t	$r_t^{\max}$	τ_2	G_t	τ_3	*%ex.*	q_t	p-value	$1-p_t$
AT&T (ATT) Jan 1996 - Apr 2006																
*17 Apr 98	10307639	1.03	29	2178	2442	97.70%	−20484	2963	441%	9	1,605,881	21	100%	0.014	0.022	0.998
*25 Apr 00	10667683	1.04	25	14673	8512	99.50%	9847	12786	593%	10	9,407,938	19	100%	0.002	0.021	0.998
*26 Apr 00	10667683	1.02	24	23185	2637	93.90%	3422	1853	447%	9	2,348,288	15	100%	0.038	0.002	0.998
Coca Cola (KO) Jan 1996 - Apr 2006																
*24 Aug 98	10423228	1.00	26	4338	2134	94.50%	5285	3007	577%	9	2,246,363	6	100%	0.034	0.000	0.998
*26 Aug 98	10423228	0.99	24	7033	1439	88.90%	2910	1792	547%	7	1,381,344	4	100%	0.048	0.015	0.998
*18 Mar 99	11199798	0.98	30	1320	1902	93.10%	993	2082	175%	10	616,950	21	100%	0.006	0.000	0.998
*23 Aug 00	10973464	1.07	59	48	2257	96.10%	4890	2258	208%	7	698,259	17	100%	0.002	0.004	0.998
12 Feb 01	11851575	1.01	96	8130	756	72.80%	1060	759	166%	9	665,280	26	100%	0.012	0.117	0.996
20 Feb 01	20207914	0.97	25	945	1796	93.10%	3153	2349	254%	10	1,340,364	19	100%	0.042	0.248	0.998
28 Jun 02	20556780	1.12	50	12516	4664	98.70%	6891	5130	312%	10	1,935,470	17	100%	0.010	0.100	0.998
*9 Jul 02	20556781	1.03	39	4755	2659	97.30%	8167	3243	669%	9	789,515	29	100%	0.016	0.000	0.998
*10 Jul 02	20703870	0.99	10	5514	3013	97.70%	4528	5533	641%	8	779,200	4	100%	0.022	0.002	0.998
Hewlett Packard (HPQ) Jan 1996 - Apr 2006																
*14 May 98	10552311	1.00	37	2646	2745	96.90%	9720	4943	117%	10	1,470,119	13	100%	0.026	0.000	0.998
15 Sep 99	10087563	1.21	66	1785	1554	93.90%	4079	1917	200%	7	1,501,894	26	100%	0.022	0.344	0.998
*15 Oct 99	10848801	0.97	36	3403	6194	99.30%	−12522	7732	130%	9	1,277,513	4	100%	0.004	0.026	0.998
*28 Sep 00	11163103	0.97	23	2600	1220	85.90%	1449	1353	271%	10	1,166,625	3	100%	0.032	0.000	0.998
*30 Oct 00	11136235	0.96	19	5307	11513	99.90%	66131	5898	118%	10	4,178,669	15	100%	0.002	0.000	0.998
*31 Oct 00	10519981	1.16	18	0	13093	99.90%	43002	295	449%	10	3,917,616	14	100%	0.002	0.000	0.998
*9 Nov 00	10373575	0.95	9	17186	4453	98.50%	6502	7170	176%	3	1,847,794	4	100%	0.012	0.000	0.998
Philip Morris (MO) Jan 1996 - Apr 2006																
28 Jan 99	11211572	1.03	23	1237	3307	92.30%	3647	3314	444%	10	2,329,156	16	100%	0.008	0.187	0.998
30 Mar 99	11439476	0.94	18	5939	20993	99.10%	43843	21330	149%	6	6,038,594	13	100%	0.002	0.160	0.998
21 Aug 00	10577641	1.07	26	3590	5770	97.90%	8428	6262	145%	10	892,463	19	100%	0.010	0.489	0.996
*16 Mar 01	20241596	0.96	36	2902	3416	93.50%	−67790	3539	122%	5	938,726	16	100%	0.014	0.020	0.998
*3 Jun 02	20705047	1.04	47	16001	15344	97.90%	14567	16767	106%	10	3,291,798	16	100%	0.016	0.005	0.998
21 Jun 02	20705047	0.96	29	43143	7298	92.10%	−82813	8816	263%	5	2,079,930	2	100%	0.048	0.211	0.998

Table 1.6. Description of detected informed trades for various sectors. For definition of entries see Page 38.

Summary of various sectors Jan 1996 - Apr 2006

Day of transaction	Market condition	Return	Crash in stock	Event's Description
			AT&T (ATT) Jan 1996 - Apr 2006	
*17 Apr 98	0.4%	−2.9%	27 Apr 98	Announcement 20 Apr 98: financial statements for first quarter
*25 Apr 00	0.7%	−19.0%	02/03 May 00	Announcement 02 May 00: financial statements for first quarter
*26 Apr 00	1.5%	−19.0%	02/03 May 00	Announcement 02 May 00: financial statements for first quarter
			Coca Cola (KO) Jan 1996 - Apr 2006	
*24 Aug 98	0.6%	−10.5%	31 Aug 98	Announcement 17 Sept 98: international crisis (Russian, Asian) hurts KO's profit
*26 Aug 98	0.0%	−10.5%	31 Aug 98	Announcement 17 Sept 98: international crisis (Russian, Asian) hurts KO's profit
*18 Mar 99	1.4%	−3.0%	31 Mar 99	Announcement 29 Mar 99: unexpected drop in sales due to Pepsi IPO
*23 Aug 00	−0.9%	−3.8%	30 Aug 00	Not identified
12 Feb 01	0.9%	−9.6%	21/22 Feb 01	Announcement 21 Feb 01: Coca-Cola/Procter&Gamble deal
20 Feb 01	−0.5%	−9.6%	21/22 Feb 01	Announcement 21 Feb 01: Coca-Cola/Procter&Gamble deal
28 Jun 02	0.1%	−3.9%	12 Jul 02	Announcement 14 Jun 02: stock options granted to executives are recorded as expense
*9 Jul 02	0.1%	−10.0%	18/19 Jul 02	Announcement 17 Jul 02: financial statements for 2nd quarter
*10 Jul 02	−0.5%	−10.0%	18/19 Jul 02	Announcement 17 Jul 02: financial statements for 2nd quarter
			Hewlett Packard (HPQ) Jan 1996 - Apr 2006	
*14 May 98	−0.7%	−13.9%	14 May 98	Announcement 14 May 98: profit warning for 2nd quarter due to Asian crisis
15 Sep 99	−0.1%	−6.2%	29 Sep 99	Announcement 01 Oct 99: fall in 4th revenues growth
*15 Oct 99	−1.0%	−12.6%	27 Oct 99	Announcement 27 Oct 99: earnings shortfall in 4th quarter
*28 Sep 00	0.7%	−12.5%	29/02 Sep/Oct 00	Not identified
*30 Oct 00	−1.8%	−12.8%	10/13 Nov 00	Announcement 13 Nov 00: financial statements for 4th quarter (ended on Oct 31)
*31 Oct 00	−2.0%	−12.8%	10/13 Nov 00	Announcement 13 Nov 00: financial statements for 4th quarter (ended on Oct 31)
*9 Nov 00	−0.5%	−12.8%	10/13 Nov 00	Announcement 13 Nov 00: financial statements for 4th quarter (ended on Oct 31)
			Philip Morris (MO) Jan 1996 - Apr 2006	
28 Jan 99	0.1%	−8.7%	10 Feb 99	Announcement 10 Feb 99: punitive damages of 81 million for smoker's death
30 Mar 99	−1.6%	−15.1%	30/31 Mar 99	Announcement 30 Mar 99: punitive damages of 51.5 million for inoperable lung cancer
21 Aug 00	0.7%	−2.6%	30 Aug 00	Not identified
*16 Mar 01	−0.9%	−4.8%	20 Mar 01	Not identified
*3 Jun 02	0.5%	−2.0%	6 Jun 02	Not identified
21 Jun 02	−1.0%	−15.8%	21/24/25 Jun 02	Announcement 21 Jun 02: investors reject stock because of litigation risk

Table 1.7. Summary of detected informed trades for various sectors. For definition of entries see Page 38.

Summary of EADS Jan 2003 - Jan 2008

Day	K	τ	OI_t	ΔOI_t	$q_t^{\Delta OI}$	ΔOI_t^{tot}	Vol_t	$r_t^{\max}$	τ_2	G_t	q_t	$1-p_t$
6 Apr 06	31	May 06	2523	2518	0.998	4988	2518	280%	29	665,073	0.004	0.998
7 Apr 06	32	June 06	4015	3855	0.998	6663	7710	269%	29	1,676,925	0.004	0.998
20 Apr 06	30	June 06	1055	1000	0.934	1545	1000	389%	22	977,515	0.016	0.998
8 May 06	30	June 06	2865	810	0.922	1920	810	1,487%	28	816,670	0.020	0.998
18 May 06	31	June 06	3040	2518	0.990	2519	2518	255%	20	1,720,467	0.008	0.996
19 May 06	26	July 06	5236	4061	0.998	−220	4061	924%	19	1,472,680	0.004	0.998

Table 1.8. Summary of detected informed trades for the case of EADS: *Day*, day of the transaction; K, strike of the selected option; τ, maturity of the selected option; OI_t, level of open interest on the transaction day; ΔOI_t, increment in open interest from day $t-1$ to day t; $q_t^{\Delta OI}$, quantile of the increment ΔOI_t from its two-year empirical distribution; ΔOI_t^{tot}, total increment in open interest; Vol_t, corresponding option volume; $r_t^{\max}$, maximum return realized within 30 days of the transaction; τ_2, day of the maximum return after the transaction; G_t, realized cumulative gains after 60 trading days due to the exercise of these options; q_t, ex-ante probability; $1-p_t$, proxy for the probability of informed trading.

Accuracy of the hedging detection method for Citigroup on 17 Dec 2001

	h_i										
	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
Percentiles											
20 20	0.051	0.052	0.077	0.089	0.094	0.124	0.151	0.193	0.227	0.277	0.306
20 40	0.046	0.058	0.079	0.106	0.116	0.174	0.196	0.235	0.287	0.290	0.299
20 60	0.051	0.063	0.070	0.100	0.131	0.156	0.157	0.210	0.210	0.265	0.282
20 80	0.069	0.072	0.072	0.075	0.076	0.076	0.076	0.077	0.095	0.125	0.180
40 20	0.055	0.057	0.064	0.087	0.117	0.124	0.168	0.185	0.198	0.207	0.223
40 40	0.053	0.055	0.090	0.096	0.147	0.158	0.167	0.182	0.219	0.239	0.272
40 60	0.056	0.064	0.081	0.120	0.125	0.159	0.183	0.218	0.253	0.284	0.298
40 80	0.041	0.104	0.188	0.190	0.201	0.231	0.254	0.265	0.282	0.291	0.306
60 20	0.051	0.052	0.059	0.078	0.098	0.102	0.161	0.180	0.198	0.200	0.217
60 40	0.049	0.066	0.070	0.098	0.119	0.125	0.136	0.161	0.161	0.249	0.253
60 60	0.051	0.051	0.062	0.065	0.065	0.065	0.097	0.114	0.125	0.126	0.138
60 80	0.050	0.055	0.074	0.075	0.099	0.114	0.151	0.153	0.157	0.192	0.208
80 20	0.049	0.088	0.131	0.147	0.153	0.156	0.166	0.178	0.189	0.195	0.210
80 40	0.049	0.056	0.063	0.075	0.116	0.136	0.158	0.179	0.183	0.192	0.195
80 60	0.049	0.071	0.085	0.085	0.092	0.100	0.110	0.136	0.150	0.183	0.183
80 80	0.033	0.070	0.070	0.080	0.084	0.099	0.099	0.099	0.151	0.154	0.231

Table 1.9. Entries are the probabilities of rejecting the hypothesis H_0 of no hedging when informed trades occur for the Citigroup stock on day i = December 17th, 2001, i.e. $\mathbb{A}(h_i)$ in (1.4), for various levels of h_i and $\mathbf{X}_i$. h_i is the ratio between volume due to hedging and volume due to non-hedging. $\mathbf{X}_i = (|r_i|, V_{i-1}^{\text{buy,non-hedge}})$ are the conditioning variables, i.e. stock return on day i and buyer-initiated volume due to non-hedging on day $i-1$, respectively. *Percentiles* are the levels of percentiles for the distributions of $|r_i|$ and $V_{i-1}^{\text{buy,non-hedge}}$, respectively, used as values of the conditioning variables in (1.4).

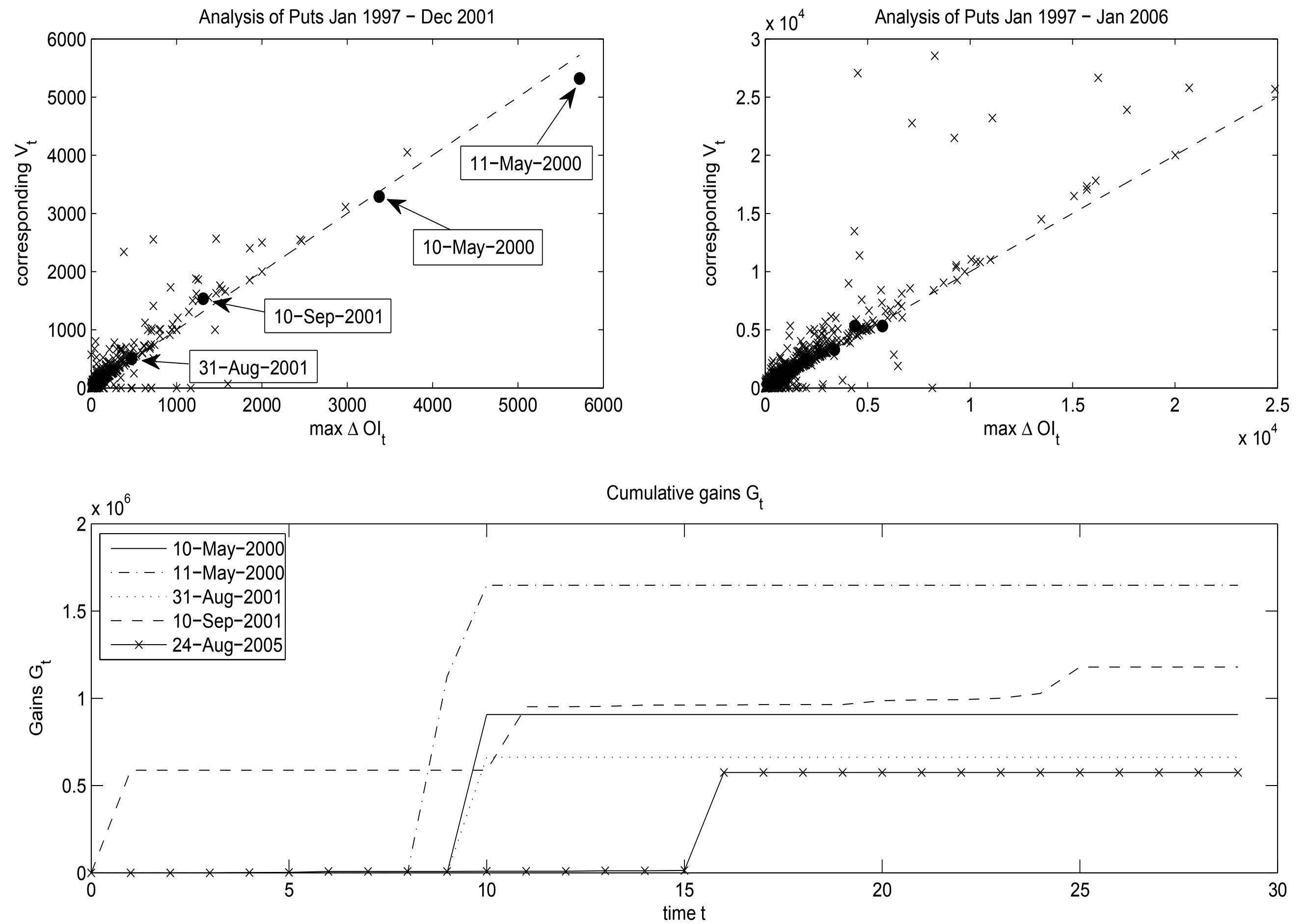

Figure 1.1. Upper graphs: Increment in open interest and volume of various put options with underlying American Airlines (AMR). Lower graph: Cumulative gains, G_t, in USD for detected informed trade options on AMR. Gains correspond to those realized by exercising the options (daily drop in open interest).

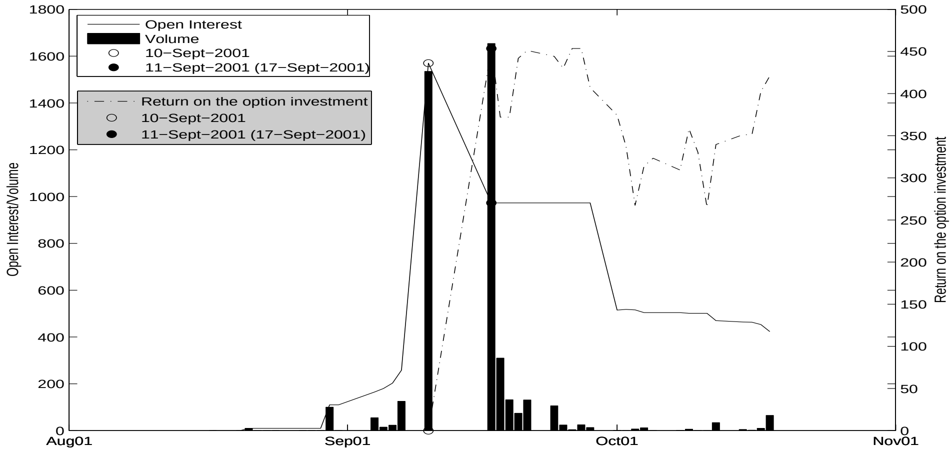

Figure 1.2. Selected put option for informed trading with underlying stock American Airlines (AMR) in the days leading up to the terrorist attacks of September 11th, 2001. The solid line shows the daily dynamic of open interest, the bars show the corresponding trading volume (left y-axis) and the dash-dot line the option return (right y-axis). The empty circle is the day of the transaction, the filled circle (partially covered by the highest bar) is the day when the market reopened after the terrorist attacks. This put option had a strike of $30 and matured at the end of October 2001.

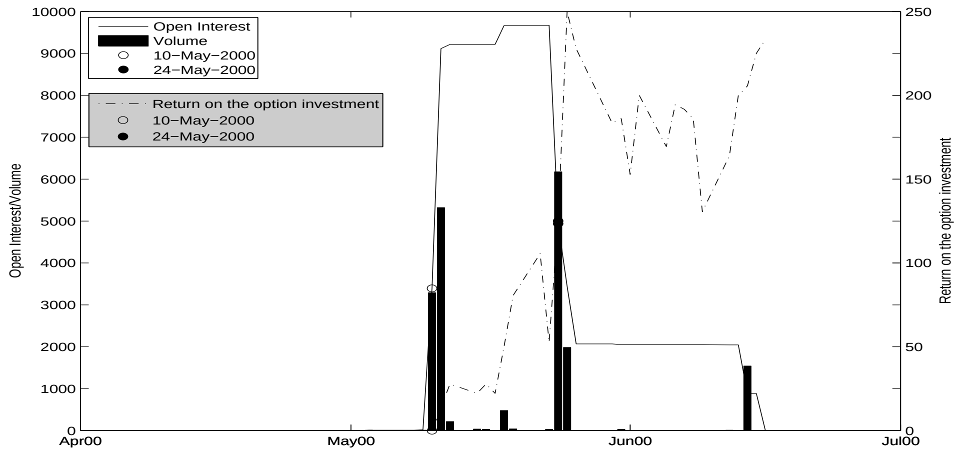

Figure 1.3. Selected put option for informed trading with underlying stock American Airlines (AMR) before the United Airlines (UAL) announcement of $4.3 billion acquisition of US Airways in May 2000. Same variables as in Figure 1.2. The empty circle is the day of the transaction, the filled circle is the day of the announcement (partially covered by the highest bar). This put option had a strike of $35 and matured at the end of June 2000.

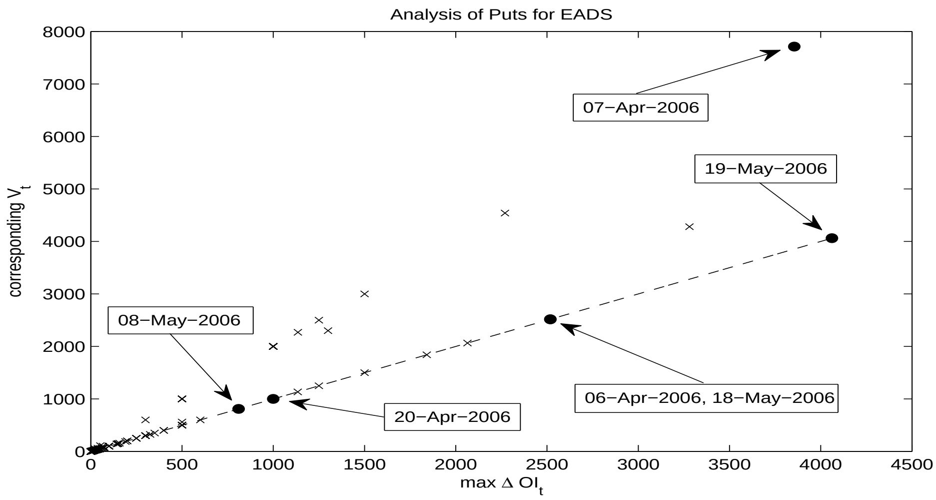

Figure 1.4. Increment in open interest and volume for various put options with underlying EADS.

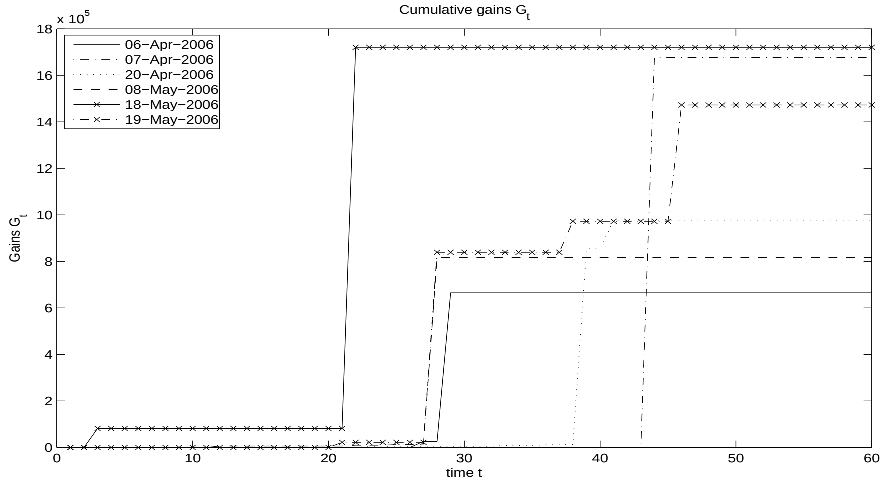

Figure 1.5. Cumulative gains, G_t, in € for detected informed trade options on EADS. Gains correspond to those realized by exercising the options (daily drop in open interest).

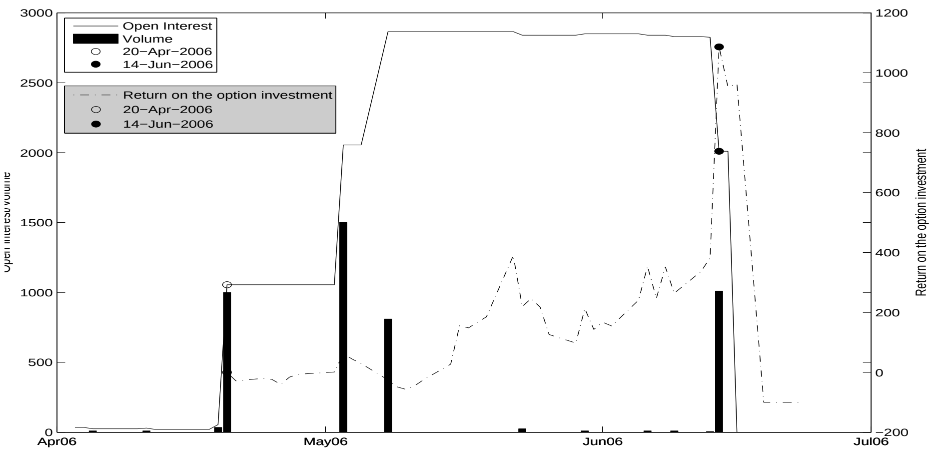

Figure 1.6. Selected put option for informed trading with underlying stock EADS before the delayed delivery announcement of the superjumbo A380 on June 14th, 2006. The option trade takes place on April 20th, 2006. The solid line shows the daily dynamic of open interest, the bars shows the corresponding trading volume (left y-axis) and the dash-dot line the option return (right y-axis). The empty circle is the day of the transaction, the filled circle is the announcement day, June 14th, 2006. This put option had a strike of €30 and matured at the end of June 2006.

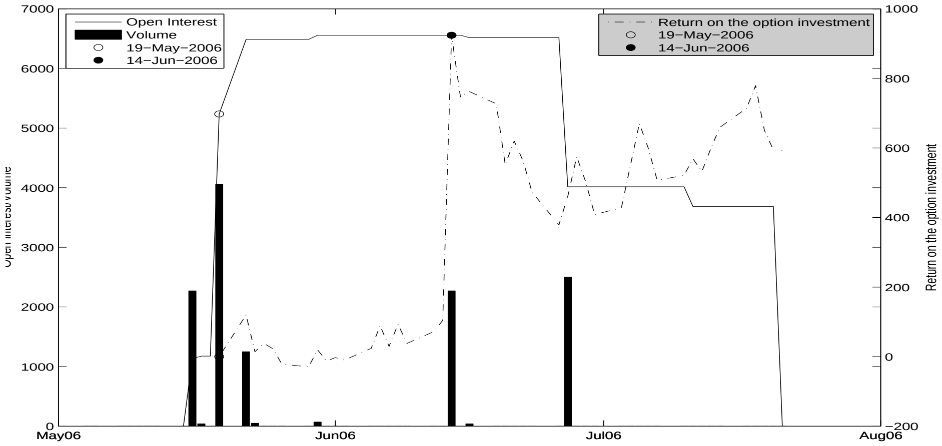

Figure 1.7. Selected put option for informed trading with underlying stock EADS before the delayed delivery announcement of the superjumbo A380 on June 14th, 2006. The option trade takes place on May 19th, 2006. Same variables as in Figure 1.6. This put option had a strike of €26 and matured at the end of July 2006.

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Informed Trading Activities in the Options Markets during the Financial Crisis

Marc Chesney, Remo Crameri, Loriano Mancini

Summary. Option trading strategies with underlying financial and insurance institutions strongly affected by the ongoing financial crisis are analyzed. We explore three various options markets: the Chicago Board Options Exchange (CBOE), with companies such as AIG, Lehman Brothers, Bear Stearns, Fannie Mae and Freddie Mac, among others; Eurex (Zurich and Frankfurt), with United Bank of Switzerland, Credit Suisse Group and Deutsche Bank; and Euronext (Paris and London), with Societé Générale, BNP Paribas and HSBC. Our empirical findings suggest that periods leading up to key events such as the takeovers of AIG and Fannie Mae/Freddie Mac, the collapse of Bear Stearns Corporation and public announcements relating to large losses/write-downs are preceded by profitable trading activities in put and call options. The realized gains amount to several hundreds of millions of dollars.

Keywords: Put/Call Options, Open Interest, Informed trading, Financial Crisis

JEL Classification: G12, G13, G14, G17, G34, C61, C65

2.1 Introduction

Despite the catastrophic implications associated with the ongoing financial crisis (2007 - present), the behavior of financial markets over the past three years has presented an interesting background for academic research in many areas due to the huge losses/write-downs announced in the financial press, and the unprecedented measures adopted by many governments. Bailout programs are one type of measure carried out in the attempt to save institutions from bankruptcy or insolvency. The term bailout has only recently acquired prominence and is not yet even listed in the Oxford English Dictionary. In the following paper the term bailout is used to denote the act of providing an institution with capital in order to prevent bankruptcy, insolvency or liquidation. As a result of the ongoing financial and economic crisis, the United States Department of the Treasury announced its voluntary Capital Purchase Program designed to encourage U.S. financial institutions to increase their capital in order to strengthen the U.S. economy by increasing the flow of financing to U.S. businesses and consumers. The program, under which the Treasury was willing to purchase up to $250 billion in senior preferred shares, was available to qualifying U.S. controlled banks and savings associations. According to the terms of the program, each financial institution was able to obtain a maximum of $25 billion Tier 1 capital. Along with the obligation to pay an interest rate as high as 5% per annum during the first five years and 9% p.a. thereafter, each financial institution willing to participate in the program had to adopt the Treasury Department standards for executive compensation and corporate governance for the period during which the Treasury holds equity issued under the program . Due to the unprecedented volume of the Capital Purchase Program and the huge losses/write-downs seen over the past two years, stocks of many companies have been subject to astonishing ups and downs as well as huge amounts of equity value erased over a remarkably short period of time. It might have been tempting for informed agents to exploit private information concerning default risk and bailout programs before its public release in order to take advantage of those large stock movements.

Due to the leverage effect offered by the options market, these robust movements in the underlying asset might have generated large gains if a suitable trading strategy using put/call options had been carried out. In this paper we aim primarily to detect such profitable informed trading activities based on options strategies. We discuss and analyze data from, among others, financial institutions that have obtained several billions under the terms of the program, and other companies severely affected by the financial crisis. Three different options markets are explored: the Chicago Board Options Exchange (CBOE), with companies such as AIG, Lehman Brothers, Bear Stearns, Fannie Mae and Freddie Mac, among others; Eurex (Zurich and Frankfurt), with United Bank of Switzerland, Credit Suisse Group and Deutsche Bank; and Euronext (Paris and London), with Societé Générale, BNP Paribas and HSBC.

Numerous financial instruments can be used to exploit trading strategies based on private information. In the following, we briefly present two cases reported in the financial press in which stocks and CDOs are used as trading instruments.

The first case was reported in the Wall Street Journal on Tuesday, April 15, 2010. It stated that prosecutors were examining whether Goldman Sachs Group Inc. director Rajat Gupta gave inside information about the bank to Galleon Group hedge fund founder Raj Rajaratnam. It was believed that Raj Rajaratnam and others used private information to trade the banks' shares at the height of the financial crisis. Mr. Rajaratnam, one of America's most successful technology investors, was one of 21 people charged in their alleged role in two overlapping groups that were thought to have made $50 million from illegal share trading. U.S. Attorney's office prosecutors said they believed that Mr. Rajaratnam and some co-conspirators traded shares in at least 22 companies, rather than in the 12 initially named in the case. In a March 22, 2010 letter filed with a New York court, prosecutors wrote that evidence from wiretaps and intercepted and recorded phone calls showed that the hedge fund boss and others executed securities transactions on the basis of nonpublic information on at least 22 companies. The letter said that Goldman Sachs shares were traded in or about June 2008 through in or about October 2008, when the worst of the financial crisis caused the bank's stock to plunge.

The second case was announced in the financial press on Monday, April 19, 2010 and is known under the name of Abacus mortgage-backed CDOs. The Goldman Sachs Group, Inc. was charged with deceiving clients by selling them mortgage securities secretly designed by a hedge fund firm run by John Paulson, who made a fortune betting on the housing market's collapse. Regulators say Goldman allowed Mr. Paulson's firm, Paulson & Co., to help design CDOs created from a specific set of risky mortgage assets, essentially setting up the CDOs for failure. Paulson then bet against it while investors in the CDOs weren't told of Paulson's role or intentions. According to the Guardian dated Monday, April 19, 2010[1], Paulson made $20bn by piling into credit default swaps against mortgages, effectively insurance policies that would pay out if homeowners defaulted. His fund made $15bn in a single year, $4bn of which Paulson took for himself.

Interestingly, we couldn't find any article in the financial press reporting suspicious activities relating to options trading despite their appealing nature as trading instruments. The purpose of the following paper is therefore to analyze trading activities using put and call options during the ongoing financial crisis. We concentrate on banks and insurance companies which have been severely affected by the crisis. Their daily returns time-series are characterized not only by unusually large negative values, but large rises as well. Quarterly results, write-downs, bank run and sold off of stocks easily lead to daily returns of more than $\pm 20\%$. Due to the symmetry in large stock movements, we apply the procedure for detecting informed trading activities using put options developed in [1], and extend it to call options. In the following we briefly summarize the detection procedure while referring to the original paper for details. Obviously our study does not constitute proof of illegal activities. Legal proof would require trader identity and motivations, information which is not contained in our database. Therefore, whenever we refer to informed trades, we think of *suspicious* trading activities.

The paper is organized as follows: in Section 2.2 we summarize the detection procedure developed in [1] and extend it to call options. In Section 2.3 we describe the data used and Section 2.4 shows our empirical results. Section 2.5 concludes.

2.2 Detecting option informed trading activity

A suspicious informed trade in put/call options is defined using three different criteria: C_1) an aggressive trade in an option contract, C_2) which is made a few days before the occurrence of a specific event and generates large gains in the following days, and C_3) the position is not hedged in the stock market and not used for hedging purposes. These three characteristics, $C_i, i = 1, 2, 3$, lead to the following method which detects informed trading activities: first, on each day we identify the put/call option contract with the largest

[1] $http : //www.guardian.co.uk/business/2010/apr/19/bear - stearn - spurned - paulson - deal$

increment in open interest relative to its volume, we then calculate the rate of return and gain generated by this transaction through exercise, and finally, we study whether hedging demands were at the origin of the trades. Option trades which are delta hedged are not regarded as suspicious informed trades.

2.2.1 The first criterion: Increment in open interest relative to volume

For every put/call option k available at day t we compute the difference $\Delta OI_t^k := OI_t^k - OI_{t-1}^k$, where OI_t^k is its open interest at day t. In the case that the option does not exist at time $t-1$, its open interest is set to zero. Since we are interested in unusual transactions, only the option with the largest increment in open interest is considered

$$X_t := \max_{k \in K_t} \Delta OI_t^k \tag{2.1}$$

where K_t is the set of all put/call options available at day t. Let V_t denote the trading volume corresponding to the put option selected in (2.1). We focus on transactions for which the corresponding volume nearly coincides with the increment in open interest. The positive difference $Z_t := (V_t - X_t)$ provides a measure of how often the newly issued options are exchanged: the smaller the Z_t, the less the new options are traded during the day on which they are created. In that case the originator of such transactions is not interested in intraday speculations, but has reasons for keeping her position for a longer period, possibly waiting for the realization of future events.

This first criterion already allows us to identify single transactions as potential candidates for informed trading activities. Let q_t denote the *ex-ante* joint historical probability of observing larger increment X_t in open interest and lower values of Z_t

$$q_t := \mathbb{P}[X \geq X_t, Z \leq Z_t] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\{X_i \geq X_t, Z_i \leq Z_t\}}$$

where N represents the length of the estimation window, e.g. $N = 500$ days, and $\mathbf{1}_{\{A\}}$ is the indicator function of event A. By construction, low values of q_t suggest that these transactions are unusual.

2.2.2 The second criterion: Relative return and realized gain

The second criterion takes into consideration the ex-post relative returns and realized gains from transactions with a low ex-ante probability q_t. Due to the non-availability of the signed volume in our database, we are not able to identify gains realized by selling options. We concentrate therefore on gains made through exercise, the latter reflected by a decrease in open interest. Let R_t denote the maximum return generated in the following two trading weeks, for puts

$$r_t^{\text{put}} := \max_{j=1,\dots,10} \frac{P_{t+j} - P_t}{P_t} \tag{2.2}$$

where P_t denotes the price of the selected option at day t, and similarly for calls

$$r_t^{\text{call}} := \max_{j=1,\dots,10} \frac{C_{t+j} - C_t}{C_t}, \tag{2.3}$$

with C_t being the price of the selected option.

When r_t^{put} resp. r_t^{call} is unusually high, an unusual event occurs during the two trading weeks. In the case of put options, the stock sharply decreases. For call options, a strong increase in the underlying's value takes place.

For the computation of realized gains we use decrements in open interest. Whenever the daily change in open interest of a specific option k, ΔOI_t^k, is negative, at least an amount of $|\Delta OI_t^k|$ options were exercised.

Let G_t denote the corresponding cumulative gains realized through the exercise of put options

$$G_t^{\text{put}} := \sum_{\tilde{t}=t+1}^{\tau_t} [(K - S_{\tilde{t}})^+ - P_t] \cdot (-\Delta OI_{\tilde{t}}) \cdot \mathbf{1}_{\{\Delta OI_{\tilde{t}}<0\}}$$

resp. call options

$$G_t^{\text{call}} := \sum_{\tilde{t}=t+1}^{\tau_t} [(S_{\tilde{t}} - K)^+ - C_t] \cdot (-\Delta OI_{\tilde{t}}) \cdot \mathbf{1}_{\{\Delta OI_{\tilde{t}}<0\}}$$

where τ_t is such that $t < \tau_t \leq T$, with T being the maturity of the selected option. Time τ_t is defined for put as well as call options as follows

$$\tau_t^* := \arg \max_{l \in \{t+1,\dots,T\}} \Big\{ \sum_{\tilde{t}=t+1}^{l} (-\Delta OI_{\tilde{t}}) \cdot \mathbf{1}_{\{\Delta OI_{\tilde{t}}<0\}} \leq X_t \Big\}$$
$$\tau_t := \min(\tau_t^*, 30)$$

giving the informed trader no more than 30 days to collect her gains. Generally, the sum of negative decrements till time τ_t in the curly brackets will be smaller than the observed increment X_t. In that case, we add to G_t the gains realized through the fraction of the next decrement in open interest. Hence the sum of all negative decrements in open interest considered will be exactly equal to the increment X_t. Calculating G_t for each day t and each option in our database provides information on whether or not option trades with a low ex-ante probability q_t generate large gains through exercise.

2.2.3 The third criterion: Hedging option position

Option trades for which the ex-ante probability q_t is small cannot be immediately classified as exclusive information vehicles: it can be the case that such transactions were hedged by traders using the underlying asset. Without knowing the exact composition of each trader's portfolio, it is a delicate step to assess whether a transaction with a high increment in open interest is hedged or not. For days having a small ex-ante probability q_t, we attempt to assess indirectly whether these unusual trades in put options are actually delta hedged using the underlying asset. The idea is to compare the total number of shares bought for non-hedging purposes and the total volume of buyer-initiated transactions in the underlying stock. If the latter is significantly larger than the former, then it is likely that some of the buyer-initiated trades occur for hedging purposes. In the opposite case we conclude that the new option positions are naked. Following the model developed in [1], we formally test the hypothesis, H_0^t, that hedging does not take place at day t. Whenever the observed V_t^{buy} is large enough, i.e. above the 95% quantile of the predicted distribution of $V_t^{\text{buy,non-hedge}}$, it is likely that a fraction of V_t^{buy} is bought for hedging purposes. Hence we reject H_0 at day t when

$$V_t^{\text{buy}} > q_{0.95}^{V_t^{\text{buy,non-hedge}}},$$

where $q_{\alpha}^{V_t^{\text{buy,non-hedge}}}$ is the α-quantile of the predicted distribution $V_t^{\text{buy,non-hedge}}$ estimated, e.g., using the last two years of data (we refer to the original paper [1] for a more technical description of the third criterion).

2.2.4 Detecting option informed trades combining the three criteria

In this paper we apply the second method developed in [1], which uses information available before and after a given transaction. The implementation of the first method is straight forward, but not considered in this paper.

Let k_t denote the selected informed trade at day t in option k. The second method can be succinctly described using the following sets of events

- *Ex-ante criteria C_1 and C_3:*

  $\Omega_1 := \{k_t \text{ such that } q_t \leq 5\%\}$

  $\Omega_2 := \{k_t \text{ such that } H_0 : \text{non-hedging, not rejected at day } t\}$

- *Ex-post criterion C_2:*

  $\Omega_3 := \{k_t \text{ such that } r_t^{\text{put/call}} \geq q_{0.90}^{r_t^{\text{put/call}}}\}$

  $\Omega_4 := \{k_t \text{ such that } G_t \geq q_{0.98}^{G_t}\}.$

The selected informed trade belongs to all four sets, i.e. $k_t \in \Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4$. The empirical quantiles at day t of $r_t^{\max}$ and G_t distributions, $q_{0.90}^{r_t^{\max}}$ and $q_{0.98}^{G_t}$, are computed using the last two years of data.

2.3 Data

We analyze several American and European companies from the banking and insurance sectors. Various databases are used in the empirical study. For American companies, options data are from the Chicago Board Options Exchange (CBOE) as provided by OptionMetrics. Stock prices are downloaded from OptionMetrics as well to avoid non-synchronicity issues, and are adjusted for stock splits and spin-offs using information from the CRSP database. Intraday transaction prices and volumes for each underlying stock price are provided by the NYSE's Trade and Quote (TAQ) database. This database consists of several millions of records for each stock and is essential in the classification of volumes in buyer and seller-initiated trades in order to complete the analysis related to the third criterion. For European companies, options data as well as intraday transaction prices and volumes for the underlying stock are obtained from EUREX provided by Deutsche Bank, and from EURONEXT provided by NYSE Euronext database. All datasets include the daily cross section of available put and call options for each company and intraday data for the underlying assets from January 1996 to September 2009. We eliminate obvious data errors such as open interest reported at zero for all existing options by excluding those days from our analysis. The list of American companies includes (in alphabetical order): American International Group (AIG), Bank of America Corporation (BAC), Bear Stearns

Corporation (BSC), Citigroup (C), Fannie Mae (FNM), Freddie Mac (FRE), Goldman Sachs (GS), JP Morgan (JPM), Lehman Brothers (LEH), Merrill Lynch (MER), Morgan Stanley (MS), Wachovia Bank (WB) and Wells Fargo Company (WFC). Most of these companies belong to the list of banks which were bailed out and, in which, the American Treasury Department invested approximately $200 billion through its Capital Purchase Program in an effort to bolster capital and support new lending. Furthermore we analyze six European banks: United Bank of Switzerland (UBS), Credit Suisse Group (CS) and Deutsche Bank (DBK) whose options are traded on EUREX, and Societé Générale Corporate (GL), HSBC (HSB) and BNP Paribas Bank (BN) with options listed on Euronext. All analyzed options are American style.

2.4 Empirical results

In this section we discuss our empirical findings. We apply the procedure described and developed in the previous sections to the above-mentioned American and European companies. We reveal most of the results in tabular form, and discuss the most interesting results in detail. Some graphical representations are also given. Tables 2.1-2.13 offer an overview of the put/call options belonging to the intersection of $\Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4$ for the 13 American companies we consider, Tables 2.14-2.16 for European companies with options listed on Eurex and finally Tables 2.17-2.19 for European companies with options traded on Euronext. We report the following variables: the day on which the selected transaction in $\Omega_1 \cap \Omega_2 \cap \Omega_3 \cap \Omega_4$ took place (Day); the market condition at day t measured by the average return of the underlying stock over the last two trading weeks (M_t); the option strike (K); the option's price (P_t); the stock value (S_t); the option's time to maturity (τ); the (abnormal) increment in open interest from day $t-1$ to day t (ΔOI_t); its quantile with respect to its empirical distribution computed over the last two years (q_t); the corresponding volume (V_t); the maximum (for calls) and minimum (for puts) return realized by the underlying stock during the two-week period following the transaction day (r_t^{s}); the number of days between transaction day t and when this maximum return occurs (τ_1); the maximum return realized by the selected option during the two-week period following the transaction day (r_t^{o}); the number of days between transaction day t and when this maximum return occurs (τ_2); the gains realized through the exercise of the new option issued at time t (G_t); a short description of the event and why the stock drops/rises (Event's description). In our tables we report only the selected transactions falling into the time window 2007-2009. The total number of detected options for the whole sample period 1996-2009 is given in brackets immediately after the option specification (put or call).

One of the crucial differences with respect to the empirical findings in [1], is the sample period. In the latter work, we consider the time window 1996 and 2006. The majority of the detected transactions are related to idiosyncratic events having a major effect only on the company which is directly involved, and possibly on some close competitors. International events such as the Asian crisis and September 11 are exceptions which impacted several other sectors. Neither of these global events is however comparable with the incredible violence that has hit stock markets and financial institutions during the current financial crisis. The high volatility and strong correlation between financial institutions marking the breakout of the crisis have important implications for our empirical findings: in turbulent times, large drops/rises in the underlying stocks are more frequent than in tranquil times, making it challenging to always find a tangible explanation for them. This was not the case in [1], where a drop by more than 10% in share value was considered an unusual event. In most cases this drop in the underlying stock was significant enough that its cause was reported in the financial press such as the business section of the New York Times. In several cases detected in this empirical study, the cause of large drops/rises cannot always be identified using financial reports, even though the movements in the underlying stock exceed $\pm 10\%$. This could partially be related to investor behavior which, in view of the unexpected magnitude of the crisis, was totally irrational. Examples of such large price changes in the underlying stock can be found on Tables 2.1-2.19, on the tenth column reporting the maximum (for calls) and minimum (for puts) return realized by the underlying stock during the two-week period following transaction day (r_t^s).

In the following subsections, we separately analyze our empirical findings for the three markets. We comprehensively discuss a small number of particular cases. Additional tables, graphical representations and comments are available from the authors upon request. We start with some general remarks: although our sample period covers almost 15 trading years, the percentage of transactions that fall into the ongoing financial crisis (2007-2009) is remarkably high. This indicates that the transactions selected using our procedure are not uniformly distributed over the whole sample period. Rather, a large majority took place in recent years. There may be several reasons for this: firstly, in spite of the high volatility present in the markets, huge and frequent gain opportunities might have been generated, and the high leverage effect offered by option trading has been explored by many market participants; secondly, due to the dramatic and rapid collapse of the financial system, the number of corporate and governmental decisions made has sharply increased, giving rise to numerous potential information leakages and informed trading activities; finally, trades made before scheduled announcements could be based on speculative bets, the latter being facilitated by several rumors already present in the market. With respect to realized gains, the numbers are generally higher than the ones found in [1] during the period 1996-2006: by virtue of the leverage effect, large drops/rises in the underlying stock lead relatively quickly to net profits of more than 1 million through option trading. With respect to the option type, we find that the number of detected put trades is usually larger than the number of detected call trades.

It is important to mention that due to the high level of information flow present in the market, it can be difficult to classify informed trading activities during an intense period such as the ongoing financial crisis. Based on the model of [1], potential informed activities are detected by looking at the abnormality of several market variables. The model does not however take into account the market conditions under which a given option trade took place. One can argue for example that in bear market conditions -those often seen between 2007 and 2009- buying protective put options is a rational decision. If market conditions do not change and stock prices continue to decline, huge gains can be realized in the following days. Whether these kinds of transactions can be classified as potential informed activities is, however, questionable. We attempt to gauge this effect by computing a measure for the market condition at day t given by the average return of the underlying stock during the last two trading weeks (M_t). Unusually high put/call option trades which take place under negative/positive values of M_t are less suspicious. To analyze this last point, we emphasize the important role and informational content of option moneyness: in our empirical results, we detect several put transactions for which the variable M_t exhibits negative values. This might suggest that these transactions are not based on private information, but are a natural reaction to ongoing market conditions. We argue however that option moneyness brings a new dimension into our analysis: if the trading was mainly concentrated in deep out-of-the money (OTM) options, a reasonable suspicion still arises. We found for example several trades in put options with an OTM of more than 50% and negative market condition M_t. By definition, the exercise of those options leads to gains only if the stock value drops by more than 50% which indeed happened a few days later. Theoretically one could slightly amplify the three criteria proposed in [1] and account for these new dimensions in the detection procedure as well. This, however, is let for future research. When discussing some of the detected transactions, we describe market conditions and moneyness, as well. In the following section we analyze some of these cases in detail.

2.4.1 Trading activities on the CBOE

2.4.1.1 The case of American International Group (AIG)

We start with a concise chronological summary of key events impacting the destiny of AIG, and discuss thereafter our empirical findings. In October 2007, when the stock was at 68.59$, AIG entered a turbulent period. The company reported that its swaps portfolio lost $352 million. A month later, that figure was revised to $1.1 billion. Between early October and mid-November 2007, AIG's stock price fell 25%. In February 2008, AIG announced estimated losses of $11.5 billion, and that it had posted $5.3 billion in collateral, pushing down the stock to under 50$. In summer 2008, it was reported that the Justice Department was investigating AIG for possible criminal fraud. The UK's Serious Fraud Office would later announce its own probe. At the beginning of September 2008, when the stock was at 21.96$, AIG executives learned that the ratings agencies planned to downgrade the company's rating again. That would trigger more collateral calls, which AIG knew it could not begin to cover. Desperate negotiations to keep the company afloat – including a possible $75 billion bridge loan from Goldman Sachs and JP Morgan, both major counterparties on the credit default swaps – ensued. Tim Geithner, Head of the Federal Reserve Bank of New York, was called in. It became clear that AIG's level of exposure to its credit default swap losses was higher than anyone had yet understood. On September 16, 2008 the Federal Reserve Board announced that it would take a nearly 80% equity stake in AIG – effectively taking over the firm – and provide an $85 billion loan. On that day, AIG stock was at 3.75$.

Applying our detection procedure, we selected a total of 17 transactions in put options and 17 in call options for the whole sample period, with 7 and 2, respectively, falling into the financial crisis (Jan 2007-Sept 2009). We now discuss some particular cases in detail. Information regarding the remaining transactions can be found in the reported tables.

The first transaction in put options took place on October 5, 2007. The underlying stock was at 69.39$ and the market condition measure M_t was slightly positive (0.18%). This indicates that the market was not in decline and the strong request in new at-the-money put options with a value of $1.9 and a maturity of November 2007 (7,594 contracts, corresponding to 95.7% of the historical distribution) was hard to justify. This transaction precedes the first AIG reported losses concerning its business activities in CDS. In the following weeks, the underlying stock fell sharply, increasing the option's value to levels above $7. Many of these options were sequentially exercised which led to net gains of approximately $7 million. An interesting sequence of transactions took place in the days leading up to the takeover of AIG on September 16, 2008. On September 10, 11 and 12, large increments in new put options were observed on the CBOE. The maturities of these options were October and November 2008. The market was bearish during the preceding trading weeks (M_t was negative) and a large demand for protective put options seemed to be a plausible, rational consequence. The moneyness of the requested options gives however important details about these transactions: On September 10 for example, the stock traded at $17.50 and 23,137 new put options with strike $18 were requested on the market. These options were at-the money with a price of $3.40. The following day, the stock traded at $17.55 and 14,494 new put options with strike $8 were bought on the CBOE. These options were deep out-of-the money and therefore quite cheap ($0.69). Furthermore, on September 12, 14,249 new out-of-the money put options with strike $10 were bought on the CBOE. Their price was $1.465. The first two options matured in November 2008, whereas the latter matured at the end of September. Three trading days later, on September 16 and just one day after the collapse of Lehman Brothers, the Federal Reserve announced that it would take over AIG. The stock price dropped to $3.75, pushing those put options deep in-the-money and increasing their value to more than $14 in the first case, $5 in the second and third case. On the same day, 12,931 options of the first type, 13,924 of the second and 1,974 of the third type were exercised, leading to a net profit of more than $13 million, $6 million and $1 million, respectively. On September 17 and 18, when the underlying stock decreased further in value to \$2, a large number of these options were exercised, leading to unusually high profits. The total realized gains through exercise G_t amounted to \$24mio, \$4.5mio and \$7.9mio., respectively. Figures 2.1 and 2.2 report the dynamics of these transactions. The solid line shows the daily dynamic of open interest, the bars show the corresponding trading volume (left y-axis) and the dash-dot line, the option return (right y-axis). The empty circle is the day of the transaction and the filled circle is the announcement day, September 15, 2008.

We now discuss in detail a detected transaction in call options which took place on July 30, 2009. The stock traded at \$13.13 with the market condition M_t being at -2.21%. The market was therefore bearish and it was difficult to find plausible justification for the 2,806 new out-of-the money (strike K=15) call options requested on July 30 with a price of \$0.95. A few days later, on August 7, a quarterly profit announcement increased the stock value to \$27.14, raising the value of the call options to \$12.375. This represents a net profit of more than 1,200% in less than two trading weeks. The stock recovered quite quickly and on August 28 it reached the level of \$50.23, increasing the option's value to \$35. In the time period between July 30 and maturity, September 2009, exercise of the call options led to net gains of more than \$5.5 million. The remaining out-of-the money call option detected on August 18, shows similar behavior and the total net gains amounted to \$5.3 million. Additional information can be found in Table 2.1 and Figure 2.3.

2.4.1.2 The case of Bear Stearns Corporation (BSC)

As in the previous subsection, we give a short chronological events description and discuss thereafter our empirical results. The financial crisis began spreading more widely in August 2007 with the collapse of two Bear Stearns hedge funds which had heavily invested in subprime-related securities. On December 20, 2007 Bear Stearns posted fourth quarter losses of \$854 million after mortgage related write-downs of \$1.9 billion. It was the first quarterly loss in its 85-year history. In Spring 2008, Bear Stearns was the subject of a multitude of market rumors regarding its liquidity. Early in the week of March 10, 2008 rumors swirled around Wall Street that European firms had suspended fixed income trading with Bear Stearns. U.S. traders began to stop trading with Bear, hedge funds pulled money from prime brokerage accounts, money market funds reduced their investment in short-term Bear issued debt. The company then suffered a cash crunch. On Thursday, March 13, Bear shares fell more than 7% to $57 even as the Standard & Poor's 500 index rose 0.5%. Bear called JPMorgan, its clearing bank, to warn that it might not have enough cash to meet its obligations on Friday and needed emergency help. It also called the Securities and Exchange Commission and the Federal Reserve Bank of New York. In an evening conference call among the New York Fed, the Securities and Exchange Commission, the Fed Board of Governors and the U.S. Treasury, the SEC said Bear Stearns might file for bankruptcy the next morning. On Friday, March 14 the New York Fed, the Fed Board of Governors and the Treasury held a conference call to discuss the options. They decided to issue an overnight non-recourse loan to JPMorgan so that the bank could then loan money to Bear Stearns. The loan was intended to get Bear Stearns through to the weekend while the companies and government officials explored Bear Stearns' options and ways to contain potential damage. Bear shares fell 46 percent to $30.85. Credit rating agencies downgraded Bear Stearns debt and customers continued to pull funds to the point where Bear Stearns officials feared the bank would be insolvent by the time Asian markets opened on Sunday evening. On Sunday evening, March 16, JPMorgan announced that it would acquire Bear for about $2 a share and that the Fed would provide JPMorgan with a $30 billion loan backed by Bear assets. JPMorgan guaranteed billions of dollars in Bear trading obligations. The deal was announced just before Asian markets opened. On Monday, March 17 Bear shares started the day with a drop of nearly 90 percent to $2.86.

For the period 1999-2009, our procedure detected 16 transactions in put options and 11 in calls. 9 trades in puts and 2 in calls, respectively fall into the time period 2007-2009. We now concentrate on a series of trades in put options which took place in the days leading up to the collapse of Bear Stearns. We detected 6 large trades in put options from March 4 till March 14, most of them involving deep out-of-the money options. Since the dynamics of such trades are similar, we do not report all details for every detected transaction, but concentrate on a few examples.

On March 10, Bear Stearns stock traded at $62.30 and the market conditions measure M_t was at -0.60%. On that day, an impressive 11,757 new contracts of new put options with strike $30 and maturity end of March were created on the CBOE. Due to the deep out-the-money moneyness, these options were traded at the cheap price of $0.625. Such an increment corresponds to the 99.70% quantile of its historical distribution. The same options exhibited another unusually high increment the following day when its open interest increased by an additional 22,809 contracts. The price of the option even decreased to \$0.25 as the stock price increased slightly in value. On March 17, when the market reopened after the intense negotiations marathon between Bear Stearns, JPMorgan and the Fed, the stock dropped nearly 85% to \$2.86, increasing the value of these put option to \$25.30. Interestingly, the day of the announcement corresponds to the exercise date of 8,150 option contracts. On March 18, an additional 9,310 put options were exercised, leading to net gains of more than \$50 million. On March 12, the put option with strike \$40 and maturity April, were subject to an impressive large increment in open interest: on that day, the stock traded at \$61.58, making the option deep out-of-the money and tradable at \$1.875. On the day of the announcement, its value increased to \$35.3, resulting in a net profit of more than 1,700% in three trading days. The sequential exercise of these options over the following weeks generated net gains of more than \$6 million. Another extremely profitable trade in put options was detected on March 13: market condition M_t on that day was -1.21%, indicating that the rumors regarding Bear Stearns' liquidity already had an impact on the underlying stock, which, on that day, traded at \$57. The put option with strike 25\$ and maturity March exhibited an impressive increment in open interest of 26,219 option contracts on March 13. Its price was fairly cheap (\$0.275) due to its deep out-of-the moneyness. On March 17, its price exploded to \$20.3, and the exercise of almost 5,000 option contracts on that day had already generated gains of several millions. Five days later, when the option matured, total realized gains amounted to approximately \$50 million. Finally, our procedure detected another transaction in put options on March 14. As in the other cases, the involved put option (with short maturity March 07) was bought for a cheap price when it was deep out-of-the money and exhibited an impressive net return right after the March 17 announcement. After exercise on March 18 and 19, related gains totaled \$28 million. Additional information can be found in Table 2.3.

2.4.1.3 The case of Fannie Mae (FNM) and Freddie Mac (FRE)

For the case of Fannie Mae, our procedure detected 17 transactions in put options, 10 of which took place in the years 2007-2009, and 13 in call options, 4 of which during the financial crisis. In the case of Freddie Mac, these include 12 for puts and 15 for calls, respectively. 5 trades in puts and 6 in calls fall into the period after 2007.

On July 13, 2008, after a weekend of negotiations, the Treasury and the Federal Reserve announced emergency measures to backstop Fannie Mae and Freddie Mac. The two companies would get access to credit lines, including direct access to Fed money if necessary, and a provision for the Treasury to take an equity stake in the companies if required. The Securities and Exchange Commission announced measures aimed at stemming the spread of false rumors. Two days later, Fannie Mae and Freddie Mac shareholders still found no overt assurance regarding the fate of common stock in any government bailout. Freddie Mac shares plunged 26% and Fannie Mae plummeted 27%. In the following days, Freddie Mac completed its second successful debt sale of the week, and confidence regarding the fate of the rescue effort moving through Congress rose. Fannie Mae shares rose more than 18% and Freddie Mac added nearly 22%. On July 23, the House of Representatives approved a housing market support package including a mandate for the U.S. Treasury to provide equity or debt to Fannie Mae and Freddie Mac. The White House dropped opposition to other measures in the broad housing bill and pledged to sign it into law. Fannie Mae shares rose almost 12% to $15, their highest close since July 9. Freddie closed up more than 11% at $10.80, its highest close since July 8. On August 8, Fannie Mae posted a second quarter loss of $2.3 billion before preferred dividend payments, or $2.54 a share. It was the fourth straight quarterly loss, bringing its cumulative loss over 12 months to $9.44 billion before preferred dividends. Fannie cut its dividend and said it would raise loss reserves. Based on an article published on August 17 in Barron's magazine, the Treasury Department was increasingly likely to recapitalize Fannie Mae and Freddie Mac in the coming months using taxpayer's money. The following day, share prices for mortgage finance companies dropped, with Fannie Mae's price plunging 22% to a 16-year low of $6.15 and Freddie Mac's down 25% to $4.39. The New York Times and Washington Post newspapers reported late on Friday, September 5, that in what could be the largest financial bailout in the nation's history, the U.S. government planned to put government sponsored mortgage finance companies Fannie Mae and Freddie Mac under federal control. The closing share price on that Friday was $7.04 for Fannie Mae and $5.1 for Freddie Mac. On Sunday, September 7, 2008 the Federal Government announced its takeover of Fannie Mae and Freddie Mac, effectively nationalizing them. At that point Fannie Mae and Freddie Mac owned or guaranteed about half of the U.S.'s $12 trillion mortgage market. This led to panic as almost every home mortgage lender and Wall Street bank relied on them to facilitate the mortgage market; investors worldwide owned $5.2 trillion of debt securities backed by them. On Monday, September 8, when the market reopened, the stock price of Fannie Mae crashed by almost 90% to under $1, and Freddie Mac stock fell to $0.88, decreasing its value by more than 80%.

Our procedure detected a series of transactions in put options starting on August 11, the month leading up to the takeover of Fannie Mae and Freddie Mac. 6 abnormal increments in put options were found for the underlying Fannie Mae and one for Freddie Mac. In all cases, the acquired put options were deep out-of-the money, making them available at a cheap price. On September 7, when both underlying stocks lost more than 80% of their value, these options went deep in-the-money and, through a sequential exercise in the following days, several millions in net gains were collected. We now discuss a few of these transactions in detail. Additional information for the remaining ones can be found in Tables 2.5 and 2.6 and Figures 2.5 and 2.6.

For Fannie Mae, on August 11, 2008 the put option with strike $6 and maturity September saw an impressive increment in open interest of 10,164 contracts. The underlying stock traded at $8.40, the market condition variables were slightly negative (-0.21%) and the put price was $0.675. Before this strong increment, the level of open interest was almost zero. In the following weeks, the open interest of these options continuously increased, reaching a maximum number of 31,824 contracts on September 4, where another strong increment of 5,774 contracts was detected by our procedure. On that day, the price of the underlying stock was $6.42 and the price of the put option $0.75. On Monday, September 8, the day after the announcement that the Fed would take over Fannie Mae and Freddie Mac, the value of the put options increased by more than 600%, reaching a value of $5.3 per option contract. On the same day, 7,162 contracts were exercised, leading to net gains of more than $3 million. Furthermore, another large number of options (11,730 contracts) were exercised a few days later. The net gains from this exercised amount were more than $5 million. Another put option with underlying stock Fannie Mae was heavily traded on August 28. Figure 2.5 provides a graphical representation of it. The increment in open interest totaled 15,178 contracts, the strike price was $7 and the option had a time-to-maturity of 114 days. The underlying stock traded at $7.95 and the put option had a value of $2.6. Before this large demand, its open interest was almost zero. Until September 9, the level of open interest remained constant. The day after, 14,701 contracts were exercised when the option's price was \$6.2. The net gains amounted to more than \$5 million. In the case of Freddie Mac, our methodology detected only one transaction in put options on September 3, 2008. Its strike was \$3 with a time-to-maturity of 136 days. The underlying asset traded at \$5.38 and the put had a value of \$0.9. The strong increment in open interest observed on September 3 (2,260 contracts) was offset by the exercise of 2,430 options on September 10, when the option had a value of \$2.35. The net gains from this transaction amounted to approximately \$300,000. With respect to the detected transaction in call options, both companies were subject to heavy trade in calls in March 2008. Our procedure detected three trades for Fannie Mae on March 5, 7 and 11, and three for Freddie Mac on March 10, 11 and 18. All these options were almost at-the-money and matured at the end of March. Interestingly, the market condition variable was between -1% and -2%, indicating that these call options had been bought during a bearish period. We do not provide the details of these transactions in this paper. They are, however, available upon request from the authors. The dynamics behind these trades are the same as those described in the previous examples: the observed increments in open interest are all above their 94% historical quantile and occurred during the days leading up to March 20, 2008 when U.S. regulators eased capital requirements for the two firms in order to provide up to \$200 billion in immediate liquidity for stressed mortgage markets. On that day, shares of Fannie Mae and Freddie Mac jumped by approximately 26%. In both cases, a considerable number of call options were exercised in the subsequent days leading to net gains of several million. Additional details can be found in Tables 2.5 and 2.6.

2.4.1.4 Final remarks on the CBOE transactions

For the remaining companies with options traded on the CBOE, we do not report detailed results and discussions, but refer to the corresponding tables for additional information. Without going into extensive detail, we collect and mention a list of cases, which, through option trading, realized substantial gains. The dynamics of these trading strategies are the same as in the previous extensively discussed examples: a large increment in put/call is observable a few days before a specific event occurs, thereby substantially decreasing/increasing the underlying asset; due to the leverage effect offered by the options market, option prices rise and, through subsequent exercise of the options, large gains are made.

For Bank of America Corporation, on November 14, 2008 and January 14, 2009 two large transactions in put options were detected. Both trades were followed by a stock crash of more than 20% due to the announcements of 35,000 and 1,000 job cuts, respectively. The resulting gains through exercise amounted in the first case to \$5.4 million, and \$3.3 million in the second one. Quarterly profits announced on July 22, 2008 (with a stock value increase of 22.4%), were preceded by a series of transactions in call options resulting in total net gains of more than \$8 million. For Citigroup, our method detected a transaction on Friday, November 21, 2008 in a deep out-of-the money call option with short time-to-maturity. The increment in open interest amounted to 61,927 option contracts, i.e. the 99.7% quantile of its historical distribution. The market condition was at -2.41%. On the following Monday, the government's plan to help Citigroup by buying \$20 billion of preferred stock was announced. The stock value increased in the following days by more than 50% and a large number of call options were exercised, leading to net gains of more than \$7 million. The substantial losses reported in the financial press on January 16, 2009 which induced a drop in the underlying stock of more than 23%, were preceded by three transactions in out-of-the money put options traded on the CBOE on January 7, 8 and 12. The total realized gains after the stock crashed amounted to more than \$9 million. In the case of Goldman Sachs, the events leading to the (relatively small) movements in the underlying stock were difficult to identify. For this reason, we do not offer any details. The profit drop of 76% announced by JP Morgan on January 15, 2009 was preceded by three large trades in put options on December 31, January 2 and 6. On these days the market condition M_t was close to zero. Realized gains totaled more than \$17 million. The strong rise in stock value between March 9 and March 18 (from \$15.9 to \$27.11) was preceded by unusually high increments in out-of-the money call options between March 5 and 9. Realized gains from options exercise totaled more than \$16 million. For Lehman Brothers and Merrill Lynch, we refer to Tables 2.9 and 2.10 for our empirical findings. For Morgan Stanley, we found two large transactions in deep out-of-the money call options on October 9 and 10, 2008. These precede the announcement on Monday, October 13 that a Japanese bank intended to buy 1/5 of Morgan Stanley. The stock value nearly doubled that day, resulting in net gains of more than \$12 million through the exercise of those call options. An interesting series of transactions in put options with underlying stock in Wachovia Bank was detected during the month of September 2008, the period leading up to the announcement on September 29 that the bank would be taken over due to its uncertain situation. On that day, the stock plummeted by more than 81%, pushing these put options deep in-the-money. The subsequent exercise of these options led to realized gains of more than \$23 million. For the last company analyzed on the CBOE, Wells Fargo, the underlying stock had been sharply loosing value during the first two months of 2009: the stock was traded around \$30 in January 2009, and on February 27, it was worth \$12.1. We detected a large number of new issued put options during this period on January 6, 7, 8 and 28. For the first three days, the market conditions were positive, whereas for the last one, a level of -2.31% was observed, indicating the bearish market situation already mentioned. The subsequent exercise of these put options led to substantial gains.

2.4.2 Trading activities on Eurex (Frankfurt and Zurich)

Option contracts with underlying German and Swiss companies are traded on Eurex, one of the world's largest derivatives exchanges and the leading clearing house in Europe established in 1998 after the merger between Deutsche Terminbörse (DTB, the German derivatives exchange) and SOFFEX (Swiss Options and Financial Futures). In this section we use the EUREX database provided by Deutsche Bank to analyze option transactions with underlying United Bank of Switzerland (UBS), Credit Suisse Group (CS) and Deutsche Bank (DBK). Our empirical findings are summarized in Tables 2.14, 2.15 and 2.16. In the case of UBS, our procedure detected 16 transactions in put options, 13 of which fell into the period 2007-2009. The proportion of call options is smaller, with 3 out of 13 transactions taking place during the financial crisis. For CS, we detected 16 trades in puts and 13 trades in calls for the entire sample period. The proportion falling into the period after 2007 is around one third. For DBK, we identified a total of 16 transactions in put and 3 in call options. More than half of these put trades took place in the last two years of our data sample, whereas only one call was found for the years 2007-2009. In the following section we discuss some particular cases and refer to the corresponding tables for further details. Related figures are available from the authors upon request.

2.4.2.1 The case of United Bank of Switzerland (UBS)

Our detection procedure identified three trades in put options which took place in October 2007, maturing in October, December and June 08. Two of the three acquired options were out-of-the money. These trades preceded the October 30 announcement that UBS, Europe's largest bank by assets, reported its first quarterly loss in almost five years.

Declines in the U.S. subprime mortgage market led to \$4.4 billion in losses and write-downs on fixed-income securities. Third quarter net loss was 830 million Swiss francs (\$712 million). In the following weeks and months, UBS stock started an impressive decline, and through the exercise of these puts options net gains of more than CHF 24 million were collected. On February 14, 2008 UBS saw its shares fall to a four-year low after it produced the worst quarterly loss in the bank's history and revealed new details of its full exposure to the sub-prime and credit crises. Its stock fell more than 8% in Zurich and New York as executives failed to rule out further write-downs - which already totaled \$18.1 billion - or give a date for a return to profitability. The fall accelerated after U.S. Federal Reserve Chairman, Ben Bernanke, said investment banks would have further write-downs. UBS confirmed that it lost CHF 12.5 billion in the final quarter of 2007, with full-year losses of CHF 4.4 billion - the first in the decade since it merged with the Swiss Bank Corporation - and had written off \$13.7bn in the final quarter of the preceding year. Interestingly, our method was able to identify three transactions in put options on January 30, and February 11 and 12, 2008. All options had short-term maturities and generated high returns after the stock crash on February 14. Collected gains amounted to nearly CHF 7 million. With respect to call options, we identified three trades falling into the period 2007-2009, whose gains amounted to more than CHF 10 million. The rise in the underlying stock price leading to high options returns could not be seen in the financial press.

2.4.2.2 Final remarks on EUREX transactions

For Credit Suisse Group and Deutsche Bank, the remaining companies that belong to our database with options traded on EUREX, we do not report detailed results and discussions, but refer to the corresponding tables for additional information. We discuss one specific event related to CS that, due to its strong impact on the underlying asset, was reported in the financial press. On October 13, 2008 Israeli holding company Koor Industries (KOR.TA) invested CHF 1.2 billion in Credit Suisse in exchange for a 3 percent stake in the bank. On that day, CS jumped by more than 27%. Furthermore, on October 16, 2008 Credit Suisse raised approximately CHF 10 billion, or about 12 percent of its outstanding equity, from private investors. The Qatar Investment Company increased its stake in Credit Suisse to 8.9%, while Saudi conglomerate Olayan increased its stake to 3.6%. Our procedure detected a trade on September 18 in deep out-of-the money call options with maturity Dec 2008. The increment in open interest amounted to 10,010 contracts, being at the 93% quantile of its historical distribution. Due to the remarkable rise in stock value observed a few weeks later, these options went in-the-money and saw gains through exercise of approximately CHF 1.5 million. In the case of Deutsche Bank, we found several trades in put options whose gains correspond to several million. In the majority of the cases, however, it was difficult to attribute a specific event to these transactions. One exception represents the three last trades on September 18 and 19 and October 1, 2008 where the large increment in put open interest and the subsequent large drop in the underlying stock can be a consequence of the Lehman Brothers collapse which had occurred a few days earlier.

2.4.3 Trading activities on Euronext (London and Paris)

Options with underlying French and British companies are traded on Euronext in Paris and London. In this subsection we report our empirical findings for Societé Générale (GL), BNP Paribas (BN) and HSBC (HSB). We discuss some specific cases, and information regarding the remaining transactions can be found in the corresponding Tables 2.17-2.19.

2.4.3.1 The case of Societé Générale (GL)

On January 24, 2008 the bank announced that a single futures trader at Societé Générale had fraudulently lost the bank €4.9billion, the largest such loss in history. Jérome Kerviel, a relatively junior futures trader, allegedly orchestrated a series of bogus transactions that spiraled out of control amid turbulent markets in 2007 and early 2008. Executives said the trader acted alone and that he may not have benefited directly from the fraudulent deals. The bank announced it would be immediately seeking €5.5 billion Euros in financing. On Tuesday, January 22, 2008 the French stock market regulator said that it had begun a formal investigation into Société Générale. It was not clear whether the inquiry was related to the revelation that Robert Day, a member of Société Générale's Board, had sold shares in the bank worth €45 million on January 18, the day Société Générale explained that management had first been alerted to Mr. Kerviel's unauthorized trading, and two days before the bank's audit committee was informed of a planned €2.05 billion write-down linked to the bank's exposure to the United States subprime lending market. Société Générale and a spokesman for Mr. Day said in separate statements that the share sales by Mr. Day and his family's trusts occurred in several transactions from December 2007 to

January 18, 2008 during a predetermined window when directors were allowed to exercise options. Both statements said all required disclosures had been made, and ”no inside information was used in any way” with respect to these sales. Our detection procedure detected two abnormal trades in put options on January 9 and 16, 2008. Both options were out-of-the money with short maturity. Relatively large increments in open interest were observed. Their exercise led to gains of more than €1.7 million. In addition, the February 12, 2008 announcement that Société Générale planned to raise $8 billion in capital was preceded by two unusually large increments in open interest in deep-out-of the money call options. After the substantial stock rise, the exercise of these options led to a total gain of more than €9 million. Other profitable trades in put as well as call options can be found in Table 2.17.

2.4.3.2 Final remarks on Euronext transactions

For BNP Paribas and HSBC, the remaining companies that belong to our database with options traded on Euronext, we do not report detailed results and discussions, but refer to the corresponding tables for additional information (Tables 2.18 and 2.19). For BNP Paribas, we briefly emphasize a series of transactions which took place between January 14 and 18, 2008. The involved put options were deep out-of-the money with short maturity. On January 30, the announcement that quarterly profit would slump over 40% had a strong impact on the underlying asset. The exercise of these put options led to a net profit of more than €2 million.

2.5 Conclusions

In this paper we extend the model developed in [1] to call options in order to capture profitable informed trading strategies arising when the underlying stock increases in value. Option trading strategies with underlying financial and insurance institutions hardly affected by the ongoing financial crisis are analyzed. Three different option markets are considered: the Chicago Board Options Exchange (CBOE), with companies such as AIG, Lehman Brothers, Bear Stearns, Fannie Mae and Freddie Mac, among others; Eurex (Zurich and Frankfurt), with United Bank of Switzerland, Credit Suisse Group and Deutsche Bank; and Euronext (Paris and London), with Societé Générale, BNP Paribas and HSBC. We find that the detected option trades are not uniformly distributed over our sample period

(1996-2009), but that the great majority falls into the period 2007-2009. Our empirical findings suggest that periods leading up to key events such as the takeovers of AIG, Fannie Mae/Freddie Mac, the collapse of Bear Stearns Corporation and public announcements relating to large losses/write-downs are preceded by unusual trading activities in options. Profitable option strategies in put as well as call options are detected. The realized gains amount to several hundreds of millions of dollars. This paper empirically shows that despite all the catastrophic consequences of the ongoing financial crisis, the crisis has simultaneously presented profitable trading strategies to market participants. Obviously our study does not constitute proof of illegal activities. Legal proof would require trader identity and their motivations, information which is not contained in our database.

American International Group (AIG)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_t	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (17)														
05 Oct 07	0.18%	70	1.90	69.39	43	7594	95.70%	9572	-3.21%	17	181.58%	10	6,948,700	CDS loss \$1.1bn
26 Feb 08	-0.64%	50	1.73	51.42	25	18091	98.30%	26703	-6.56%	6	375.36%	10	6,621,600	10 Mar 08: \$7.8bn loss
07 May 08	0.22%	43	1.29	45.08	10	15073	97.70%	23615	-8.77%	5	258.53%	4	3,282,600	08 May 08: quarterly Loss
08 Aug 08	0.68%	20	0.66	24.87	43	16951	97.30%	33507	-18.05%	2	221.97%	10	10,386,000	05 Aug 08: quarterly Loss
10 Sep 08	-0.23%	18	3.40	17.50	38	23137	98.30%	28252	-60.79%	6	367.65%	6	24,712,000	16 Sept 08: Fed lends \$85bn to AIG
11 Sep 08	-0.35%	8	0.69	17.55	37	14494	95.10%	15335	-60.79%	5	693.48%	5	4,532,100	16 Sept 08: Fed lends \$85bn to AIG
12 Sep 08	-1.24%	10	1.47	12.14	8	14249	94.90%	52077	-60.79%	4	440.96%	4	7,910,900	16 Sept 08: Fed lends \$85bn to AIG
Call Options (17)														
30 Jul 09	-2.21%	15	0.95	13.13	51	2806	64.70%	2808	62.72%	7	1352.40%	8	5,536,000	07 Aug 09: quarterly profit
18 Aug 09	4.25%	31	1.21	24.55	32	3223	70.70%	2925	26.93%	10	1539.00%	9	5,268,100	Not identified

Table 2.1. Summary of detected informed trades for American International Group. For definition of entries see Page 80.

Content of Tables: day on which the transaction took place (Day); market condition at day t measured by the average return of the underlying stock during the last two trading weeks (M_t); option strike (K); option price (P_t); stock value (S_t); its time-to-maturity (τ); increment in open interest from day $t-1$ to day t (ΔOI_t); its quantile with respect to its empirical distribution computed over the last two years (q_t); corresponding volume (V_t); maximum (for calls) and minimum (for puts) return realized by the underlying stock during the two-week period following the transaction day (r_t^{s}); number of days between transaction day t and when this maximum return occurs (τ_1);maximum return realized by the selected option during the two-week period following the transaction day (r_t^{o}); number of days between transaction day t and when this maximum return occurs (τ_2); gains realized through the exercise of the new option issued at time t (G_t); short description of the event and why the stock drops (Event's description). In most of the cases this drop in the underlying stock is large enough that its cause is reported in the financial press such as the business section of the New York Times. The cause of a few informed trades could not be identified. In those cases the movements in the underlying stock were not significant.

Bank of America Corporation (BAC)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (16)														
19 Jun 08	-0.86%	30	3.35	28.14	58	27151	99.50%	32226	-8.09%	20	126.87%	10	8,872,400	Not identified
14 Nov 08	-1.57%	15	0.59	16.42	8	21136	97.30%	34814	-20.92%	13	541.03%	5	5,410,400	11 Dec 08: cut of 35,000 jobs
14 Jan 09	-1.15%	10	0.39	10.20	3	15774	94.50%	36500	-28.97%	6	628.21%	3	3,325,600	21 Jan 10: cut of 1,000 jobs
07 May 09	2.23%	13	0.66	13.51	9	35313	97.30%	102930	-10.20%	7	167.94%	7	2,002,100	05 May 09: needs $33.9bn
Call Options (20)														
22 Jan 08	-0.36%	40	0.53	37.39	25 3	12606	98.30%	22114	8.50%	4	900.00%	9	2,383,700	24 Jan 08: 6bn new shares
08 Jul 08	-1.70%	30	0.90	23.54	193	10641	95.50%	12181	22.41%	9	244.44%	10	1,900,100	22 Jul 08: quarterly profit
09 Jul 08	-1.73%	23	0.93	22.06	10	10076	94.30%	14985	22.41%	8	435.14%	8	3,994,300	22 Jul 08: quarterly profit
10 Jul 08	-1.69%	25	1.81	22.36	135	8486	92.10%	13883	22.41%	7	417.96%	10	1,549,400	22 Jul 08: quarterly profit
15 Jul 08	-1.20%	28	0.50	21.67	130	5044	80.30%	5761	22.41%	4	1400.00%	7	1,070,500	22 Jul 08: quarterly profit
15 Sep 08	0.30%	30	0.57	26.55	5	11070	94.70%	39933	22.56%	7	935.40%	5	7,654,900	15 Sep 08: BAC acquires MER
06 Mar 09	-1.64%	4	0.24	3.14	15	26877	97.50%	49783	27.73%	5	1442.60%	9	3,337,100	10 Mar 09: surprising Value
11 Mar 09	-1.35%	7	0.43	4.93	38	31482	97.70%	41024	27.73%	2	276.74%	9	7,809,700	10 Mar 09: surprising Value
23 Apr 09	2.12%	11	0.45	8.82	23	85737	99.70%	134900	19.31%	10	353.93%	10	9,796,900	08 May 09: find Ready Investors
28 Apr 09	1.38%	11	0.23	8.15	18	58619	99.50%	101270	19.31%	7	1344.40%	9	5,719,800	08 May 09: find Ready Investors
29 Apr 09	1.61%	10	1.06	8.68	52	35473	97.10%	44553	19.31%	6	338.39%	8	9,254,600	08 May 09: find Ready Investors
04 May 09	1.80%	10	1.91	10.38	47	35626	97.10%	46098	19.31%	3	142.78%	5	6,789,800	08 May 09: find Ready Investors
05 May 09	1.78%	13	0.41	10.84	11	33455	96.50%	91088	19.31%	2	374.07%	4	2,024,000	08 May 09: find Ready Investors
23 Jul 09	-0.31%	13	0.50	12.69	30	50593	97.90%	82085	6.72%	18	635.00%	10	5,314,400	Not identified

Table 2.2. Summary of detected informed trades for Bank of America Corporation . For definition of entries see Page 80.

Bear Stearns Corporation (BSC)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (16)														
05 Oct 07	0.64%	115	1.53	131.58	43	1776	87.10%	2231	-3.75%	4	129.51%	7	2,081,900	Not identified
09 Oct 07	0.89%	125	2.53	127.46	11	5692	99.70%	7061	-5.35%	21	220.79%	9	2,085,500	Not identified
10 Dec 07	-0.43%	105	3.80	105.75	12	3713	96.10%	5529	-5.92%	20	301.32%	10	2'312'700	Not identified
04 Mar 08	-0.19%	70	1.65	77.17	18	2570	89.70%	3374	-83.97%	12	3854.50%	10	12,086,000	15 Mar 08: bank run
10 Mar 08	-0.60%	30	0.63	62.30	12	11757	99.70%	16260	-83.97%	8	3948.00%	6	28,484,000	15 Mar 08: bank run
11 Mar 08	-0.76%	30	0.25	62.97	11	22809	99.90%	57893	-83.97%	7	10020.00%	5	55'246'000	15 Mar 08: bank run
12 Mar 08	-0.79%	40	1.88	61.58	38	4156	96.10%	6210	-83.97%	6	1782.70%	4	6,013,800	15 Mar 08: bank run
13 Mar 08	-1.21%	25	0.28	57.00	9	26219	99.90%	39624	-83.97%	5	7281.80%	3	49,898,000	15 Mar 08: bank run
14 Mar 08	-1.10%	20	2.78	30.00	8	25246	99.70%	48910	-83.97%	4	451.35%	2	28,450,000	15 Mar 08: bank run
Call Options (11)														
07 Sep 07	0.09%	110	3.85	105.37	15	4791	99.10%	6245	7.67%	16	159.74%	8	1,413,400	Not identified
20 Mar 08	-8.34%	8	1.08	5.96	30	6485	98.10%	9653	88.76%	4	262.79%	2	1,667,400	Not identified

Table 2.3. Summary of detected informed trades for Bear Stearns Corporation. For definition of entries see Page 80.

Citigroup (C)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (14)														
07 Jan 09	-0.32%	5	0.36	7.15	73	13329	80.50%	13071	-23.22%	8	588.89%	9	1,476,000	16 Jan 09: report big loss
08 Jan 09	-0.09%	6	0.57	7.16	72	24515	93.90%	27180	-23.22%	7	487.72%	8	4,534,800	16 Jan 09: report big loss
12 Jan 09	-0.73%	5	0.60	5.60	40	25233	94.90%	40328	-23.22%	5	296.64%	6	3,020,000	16 Jan 09: report big loss
Call Options (24)														
22 Jan 08	-0.77%	25	0.98	24.40	25	29208	99.30%	59560	8.03%	4	392.31%	9	4,484,700	Not identified
21 Nov 08	-2.41%	5	0.92	3.77	29	61927	99.70%	101750	57.83%	4	277.72%	5	7,566,600	23 Nov 08: cash infusion from U.S
08 Apr 09	5.99%	2	0.73	2.70	38	25031	91.10%	36978	25.00%	5	193.10%	4	2,964,100	Not identified
30 Jul 09	-0.51%	3	0.24	3.14	23	216040	99.70%	464560	10.41%	1	310.64%	8	4,327,500	Not identified

Table 2.4. Summary of detected informed trades for Citigroup. For definition of entries see Page 80.

Fannie Mae (FNM)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (17)								
08 Nov 07	-1.24%	45	2.40	49.80	44	1993	91.90%	2196	-24.83%	11	606.25%	9	2,048,600	10 Nov 07: third-Quarter Loss
09 Nov 07	-0.91%	40	1.28	49.00	43	3878	96.90%	5642	-24.83%	10	864.71%	8	1,792,500	10 Nov 07: third-Quarter Loss
20 Jun 08	-0.74%	21	1.18	23.81	29	2519	81.70%	2693	-27.34%	19	148.94%	10	1,640,700	10 Jul 08: U.S. mulls future of FNM
07 Jul 08	-1.46%	18	4.55	15.74	75	6262	95.50%	10163	-27.34%	9	140.66%	7	1,096,100	07 Jul 08: U.S. mulls future of FNM
11 Aug 08	-0.21%	6	0.68	8.40	40	10164	99.10%	10657	-26.79%	10	262.96%	8	3,442,900	Not identified
13 Aug 08	0.43%	3	0.45	7.69	157	9603	97.70%	9653	-89.63%	20	150.00%	6	1,177,600	20 Aug 08: fear of potential losses
27 Aug 08	-3.48%	3	0.43	6.48	52	7752	95.90%	11376	-89.63%	10	335.29%	8	733,000	05 Sept 08: under federal control
28 Aug 08	-2.75%	7	2.60	7.95	114	15178	99.50%	15240	-89.63%	9	142.31%	7	5,541,700	05 Sept 08: under federal control
29 Aug 08	-2.97%	3	0.40	6.84	50	5582	91.90%	6610	-89.63%	8	362.50%	6	619,600	05 Sept 08: under federal control
04 Sep 08	-1.88%	6	0.75	6.42	16	5774	92.10%	7041	-89.63%	5	640.00%	10	2,609,800	05 Sept 08: under federal control
						Call Options (13)								
20 Nov 07	-1.63%	30	3.00	28.25	32	3622	96.90%	7832	18.62%	10	200.00%	8	1,432,700	07 Dec 07: raise Capital
05 Mar 08	-0.94%	26	1.03	24.27	17	10333	99.70%	6006	27.06%	12	163.41%	10	3,001,600	20 Mar 08: reduction cushion of capital
07 Mar 08	-1.51%	24	1.13	22.77	15	3580	94.30%	2992	27.06%	10	811.11%	10	842,200	20 Mar 08: reduction cushion of capital
11 Mar 08	-1.75%	23	1.53	22.00	11	5192	97.30%	8494	27.06%	8	670.49%	8	4,251,400	20 Mar 08: reduction cushion of capital

Table 2.5. Summary of detected informed trades for Fannie Mae. For definition of entries see Page 80.

Freddie Mac (FRE)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (12)								
20 Jun 08	-0.66%	24	3.15	21.82	29	2937	89.30%	3071	-26.02%	19	201.59%	10	3,318,600	07 Jul 08: plunge on capital concerns
03 Jul 08	-1.96%	15	1.60	14.50	16	5116	94.70%	6252	-26.02%	10	500.00%	8	2,674,500	15 Jul 08: rescue plan does not convince
07 Jul 08	-1.97%	13	1.90	11.91	12	9983	97.70%	12005	-26.02%	9	273.68%	7	4,009,700	15 Jul 08: rescue plan does not convince
09 Jul 08	-2.66%	10	1.25	10.26	10	12875	98.50%	24356	-26.02%	7	280.00%	5	984,260	15 Jul 08: rescue plan does not convince
03 Sep 08	-1.36%	3	0.90	5.38	136	2260	79.90%	2686	-82.75%	6	211.11%	10	320,760	07 Sep 08: government's takeover of FRE, FNM
						Call Options (15)								
20 Nov 07	-1.24%	30	1.98	26.74	32	6159	99.90%	9854	18.84%	10	188.61%	8	187,320	20 Nov 07: third quarter $2bn loss
23 Nov 07	-1.60%	25	3.60	26.47	29	2592	97.70%	3275	18.84%	8	234.72%	10	232,670	20 Nov 07: third quarter $2bn loss
27 Nov 07	-3.01%	25	2.65	25.73	25	4318	99.70%	4945	18.84%	6	354.72%	8	1,367,300	28 Nov 07: cuts dividend in half
10 Mar 08	-1.62%	20	0.55	17.39	12	2990	95.30%	4691	26.19%	9	2218.20%	9	668,520	05 Mar 08: Fed consider rescue
11 Mar 08	-1.77%	20	1.48	20.16	11	3394	96.50%	6886	26.19%	8	764.41%	8	575,570	11 Mar 08: Carlyle fund liquidation
18 Mar 08	-1.41%	25	1.55	26.02	4	3479	96.30%	11493	26.19%	3	396.77%	3	1,569,600	20 Mar 08: reduction cushion of capital

Table 2.6. Summary of detected informed trades for Freddie Mac. For definition of entries see Page 80.

Goldman Sachs (GS)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (13)								
20 Feb 07	0.35%	220	5.05	220.94	25	6162	99.50%	7953	-4.21%	14	490.10%	10	9,286,700	Not identified
20 Jul 07	0.20%	200	5.20	205.94	29	4048	95.30%	8594	-2.53%	13	168.27%	10	2,372,800	19 Jul 07: deal for 2nd Jersey City Tower
25 Jul 07	0.39%	185	1.15	203.16	24	7434	99.50%	12329	-2.53%	10	708.70%	8	5,531,800	19 Jul 07: deal for 2nd Jersey City Tower
						Call Options (15)								
17 Mar 08	-0.01%	170	0.85	151.02	5	4089	94.50%	11055	3.42%	13	970.59%	4	1,560,800	18 Mar 08: beat expectations earnings
11 Jun 08	-1.12%	170	3.90	162.40	10	11406	99.30%	20313	16.27%	11	330.77%	7	8,896,900	Not identified
16 Jul 08	0.67%	170	4.45	172.86	3	4685	95.50%	19103	5.26%	13	184.27%	3	3,419,000	Not identified
10 Mar 09	-0.98%	105	2.71	85.28	39	4014	92.70%	7132	14.35%	14	375.05%	10	1,286,800	Not identified

Table 2.7. Summary of detected informed trades for Goldman Sachs. For definition of entries see Page 80.

JP Morgan (JPM)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (20)								
20 Jul 07	-0.04%	48	1.40	47.56	29	5128	93.90%	8345	-5.03%	17	175.00%	5	1,039,200	Not identified
01 Feb 08	0.54%	48	1.08	48.25	15	11340	98.10%	12911	-5.04%	5	365.12%	10	3,452,900	Not identified
31 Dec 08	0.13%	30	2.37	31.53	52	2852	60.10%	2859	-20.73%	15	158.44%	8	890,540	15 Jan 08: profit drops 76%
02 Jan 09	-0.04%	32	3.25	31.35	50	6953	88.10%	8218	-20.73%	14	143.08%	10	4,040,000	15 Jan 09: profit drops 76%
06 Jan 09	0.66%	30	4.08	29.88	74	40772	99.30%	43454	-20.73%	12	201.23%	10	12,084,000	15 Jan 09: profit drops 76%
11 Feb 09	0.47%	26	3.01	26.09	38	8368	88.70%	10789	-13.99%	18	132.95%	8	1,272,400	Not identified
						Call Options (12)								
23 Sep 08	0.60%	43	2.30	40.56	25	8433	93.50%	10800	16.75%	1	258.39%	8	2,061,700	Not identified
05 Mar 09	-0.76%	18	1.46	16.60	16	9604	93.10%	16918	24.67%	15	566.67%	10	1,393,400	Not identified
06 Mar 09	-0.73%	21	1.22	15.93	43	10656	94.90%	19395	24.67%	14	481.97%	9	6,178,200	16 Apr 09: anticipating profits
09 Mar 09	-0.91%	18	1.94	15.90	40	10000	93.50%	10338	24.67%	13	401.29%	8	8,694,100	16 Apr 09: anticipating profits

Table 2.8. Summary of detected informed trades for JP Morgan. For definition of entries see Page 80.

Lehman Brothers (LEH)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (8)														
29 Feb 08	-0.28%	50	2.68	50.99	22	15279	99.10%	27054	-19.13%	14	182.24%	7	2,243,500	28 Mar 08: investors sold off stocks
13 Mar 08	-1.61%	50	6.15	45.99	37	13431	98.50%	15704	-19.13%	5	219.51%	3	12,680,000	28 Mar 08: investors sold off stocks
15 May 08	0.69%	45	3.18	44.77	37	5374	84.90%	5825	-13.64%	21	193.70%	7	4,055,300	10 Jun 08: loss of 3 billion
16 May 08	0.46%	41	1.99	43.64	36	10905	96.10%	11537	-13.64%	20	207.79%	6	6,332,300	10 Jun 08: loss of 3 billion
21 May 08	-0.15%	35	1.65	39.56	31	9234	94.30%	16594	-13.64%	17	266.67%	9	4,808,800	10 Jun 08: loss of 3 billion
Call Options (12)														
17 Mar 08	-1.05%	35	3.08	31.75	5	8836	99.50%	22144	46.43%	4	343.90%	4	8,450,400	19 Mar 08: earnings
15 Jul 08	-1.31%	15	0.68	13.22	4	16938	99.30%	41781	25.95%	4	503.70%	4	5,731,200	17 Jul 08: biggest one-day rally

Table 2.9. Summary of detected informed trades for Lehman Brothers . For definition of entries see Page 80.

Merrill Lynch (MER)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (15)														
18 Jun 07	-0.09%	90	2.30	90.00	33	5532	96.90%	6221	-3.54%	18	195.65%	10	1,990,500	20 Jun 07: hedge funds losses
10 Oct 07	0.25%	70	1.50	74.94	38	2362	79.50%	2593	-7.90%	20	273.33%	8	1,040,700	05 Oct 07: $5.5bn loss
23 Oct 07	-0.34%	63	1.60	67.12	25	9120	98.90%	15002	-7.90%	11	343.75%	10	1,827,300	24 Oct 07: biggest quarterly loss
11 Jan 08	-1.19%	55	2.18	54.69	8	7375	95.70%	12695	-10.24%	7	152.87%	5	1,021,300	15 Jan 08: 21bn infusion
02 Jun 08	-0.61%	43	1.93	42.62	19	10060	95.50%	19021	-6.80%	21	268.83%	8	2,614,200	02 Jun 08: ratings cut by S&P
03 Sep 08	0.25%	23	0.19	28.33	17	14031	94.70%	18682	-19.59%	21	3018.40%	8	1,847,900	15 Sept 08: BAC acquires Merrill
Call Options (17)														
14 Jul 08	-1.13%	25	3.60	25.88	33	9133	97.50%	5413	13.41%	5	151.39%	8	618,760	raise capital
15 Jul 08	-0.97%	25	4.03	24.69	95	5408	92.30%	6169	13.41%	4	144.10%	7	779,060	raise capital

Table 2.10. Summary of detected informed trades for Merrill Lynch. For definition of entries see Page 80.

Morgan Stanley (MS)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (21)														
21 Jun 07	0.21%	85	1.25	87.29	30	7347	98.90%	10368	-14.96%	10	130.00%	4	6,868,600	22 Jun 07: Blackstone Rival Plans Own I.P.O
26 Oct 07	0.20%	60	2.20	64.78	57	4570	96.10%	6601	-7.20%	7	338.64%	9	1,523,000	07 Nov 07: write-down expected
30 Oct 07	-0.18%	65	3.70	65.49	53	3045	92.70%	3207	-7.20%	5	277.03%	7	2,333,900	07 Nov 07: write-down expected
31 Oct 07	0.16%	65	2.88	67.26	52	8100	98.50%	11093	-7.20%	4	385.22%	6	9,177,000	07 Nov 07: write-down expected
15 May 08	0.55%	45	1.35	47.71	37	8745	96.70%	9126	-8.48%	18	211.11%	7	3,535,000	02 Jun 08: ratings cut by S&P
16 May 08	0.38%	45	1.53	47.21	36	4877	91.10%	4978	-8.48%	17	175.41%	6	1,527,300	02 Jun 08: ratings cut by S&P
05 Jun 08	-0.75%	43	2.35	44.59	44	12093	98.90%	12614	-8.48%	4	191.49%	5	3,856,400	02 Jun 08: ratings cut by S&P
04 Sep 08	0.02%	37	0.95	40.34	16	10713	96.50%	12073	-24.22%	12	1568.40%	10	8,582,700	18 Sept 08: plan to merger with Wachovia
05 Sep 08	0.03%	36	1.30	41.36	43	1250	45.90%	1251	-24.22%	11	1157.70%	9	1,451,100	18 Sept 08: plan to merger with Wachovia
15 Sep 08	-0.39%	30	3.75	32.19	33	10704	96.30%	20113	-25.89%	21	216.00%	3	1,920,200	18 Sept 08: plan to merger with Wachovia
Call Options (19)														
13 Mar 08	-0.57%	45	1.40	41.60	37	13151	99.90%	21130	17.81%	6	342.86%	6	1,324,500	19 Mar 08: profits
17 Mar 08	-0.14%	40	1.00	36.38	5	4122	95.10%	8154	17.81%	4	895.00%	4	2,893,900	20 Mar 08: profits
09 Oct 08	-2.50%	18	1.33	12.45	9	26507	99.30%	7920	86.98%	5	254.72%	4	3,311,900	13 Oct 08: sell 1/5 to a big Japanese bank
10 Oct 08	-3.41%	10	2.75	9.68	8	10675	98.10%	23102	86.98%	4	336.36%	3	8,553,700	13 Oct 08: sell 1/5 to a big Japanese bank

Table 2.11. Summary of detected informed trades for Morgan Stanley . For definition of entries see Page 80.

Wachovia Bank (WB)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (12)								
01 Feb 08	0.05%	38	0.98	38.76	15	12383	99.10%	19142	-8.33%	4	305.13%	10	2,317,400	Not identified
27 May 08	-0.06%	25	2.10	24.77	53	8872	95.30%	12472	-6.76%	11	197.62%	10	679,660	Not identified
03 Jun 08	-0.92%	20	0.55	21.92	18	14629	98.70%	25778	-6.76%	6	286.36%	9	2,220,700	Not identified
08 Sep 08	-0.38%	19	1.20	18.99	12	15420	96.90%	18341	-81.60%	18	729.17%	8	4,325,800	29 Sep 08: taken over of Wachovia
09 Sep 08	-0.69%	16	1.15	16.24	11	20643	97.90%	24149	-81.60%	17	504.35%	7	20,656	29 Sep 08: taken over of Wachovia
16 Sep 08	0.04%	8	1.45	11.51	32	12138	93.90%	19833	-81.60%	12	293.10%	10	2,900,300	29 Sep 08: taken over of Wachovia
22 Sep 08	-1.72%	18	5.55	14.81	117	48400	99.70%	846	-81.60%	8	181.08%	6	12,502,000	29 Sep 08: taken over of Wachovia
25 Sep 08	1.62%	20	7.00	13.70	58	6568	83.30%	6593	-81.60%	5	158.57%	3	4,121,800	29 Sep 08: taken over of Wachovia
						Call Options (15)								
11 Jul 08	-0.86%	13	1.70	11.54	36	5282	90.70%	7417	27.51%	7	211.76%	9	1,080,400	17 Jul 08: quarterly announcement
15 Jul 08	-1.70%	10	0.35	9.08	4	15541	98.70%	29187	27.51%	5	842.86%	3	4,103,800	17 Jul 08: quarterly announcement
18 Jul 08	-3.21%	13	2.18	12.97	29	51958	99.90%	53660	27.51%	2	143.68%	4	5,007,000	17 Jul 08: quarterly announcement
21 Jul 08	-2.14%	15	1.03	13.18	26	25982	99.50%	48387	27.51%	1	329.27%	10	1,773,600	17 Jul 08: quarterly announcement
30 Sep 08	1.17%	5	0.73	3.50	53	14438	97.30%	20253	90.22%	3	168.97%	4	700,240	06 Oct 08: offer to buy Wachovia
01 Oct 08	-0.68%	5	0.68	3.55	52	21237	98.90%	35462	90.22%	2	188.89%	3	894,280	06 Oct 08: offer to buy Wachovia

Table 2.12. Summary of detected informed trades for Wachovia Bank. For definition of entries see Page 80.

Wells Fargo Company (WFC)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (14)														
27 Dec 07	-0.04%	30	1.03	30.30	23	70717	99.90%	8710	-6.10%	15	256.10%	8	19,158,000	Not identified
01 Feb 08	0.48%	33	0.83	33.65	15	8353	95.30%	13513	-6.72%	4	281.82%	6	1,497,000	Not identified
04 Nov 08	-0.31%	37	3.30	35.11	18	6130	78.50%	6424	-18.97%	21	190.91%	7	4,824,000	10 Nov 08: sells 11bn in stock
06 Jan 09	0.82%	28	3.95	27.54	46	78828	99.90%	87010	-23.82%	12	259.49%	10	66,232,000	28 Jan 09: loss of fourth quarter
07 Jan 09	0.49%	28	4.80	25.87	45	24733	97.50%	25940	-23.82%	11	195.83%	9	21,451,000	28 Jan 09: loss of fourth quarter
08 Jan 09	0.26%	25	3.20	25.72	44	53561	99.30%	64303	-23.82%	10	251.56%	8	28,476,000	28 Jan 09: loss of fourth quarter
28 Jan 09	-2.31%	19	2.20	21.19	52	5080	67.30%	5540	-14.22%	12	115.91%	7	294,980	28 Jan 09: loss of fourth quarter
Call Options (15)														
18 Jan 08	-0.60%	25	1.70	25.48	29	7419	97.50%	9104	9.02%	5	423.53%	9	2,878,300	Not identified
07 Jul 08	-0.49%	28	1.10	23.52	103	11313	97.50%	13990	32.77%	10	165.91%	9	156,000	17 Jul 08: biggest one-day rally
15 Jul 08	-0.36%	23	1.03	20.51	32	22712	99.50%	29515	32.77%	4	690.24%	6	16,583,000	17 Jul 08: biggest one-day rally
10 Mar 09	-3.28%	11	2.70	11.81	39	13573	92.90%	15588	23.87%	12	159.26%	10	8,697,000	09 Apr 09: bank predicted record profit
16 Mar 09	-1.05%	18	0.75	13.70	33	5524	77.70%	5030	31.70%	21	193.33%	6	440,250	10 Apr 09: bank predicted record profit
01 Apr 09	1.90%	20	0.88	14.48	45	35308	97.10%	42665	31.70%	9	165.71%	7	3,895,600	11 Apr 09: bank predicted record profit
21 Apr 09	1.16%	20	2.23	18.81	60	12849	91.10%	13257	23.66%	12	156.18%	10	166,010	08 May 09: ready investors
23 Apr 09	1.55%	23	1.08	20.09	23	19491	95.30%	39358	23.66%	10	337.21%	10	4,108,900	08 May 09: ready investors
28 Apr 09	1.48%	18	3.85	19.48	53	5494	73.10%	5690	23.66%	7	163.64%	9	256,750	08 May 09: ready investors
30 Apr 09	1.80%	25	0.23	20.01	16	15771	93.90%	21883	23.66%	5	1366.70%	7	618,110	08 May 09: ready investors

Table 2.13. Summary of detected informed trades for Wells Fargo Company . For definition of entries see Page 80.

United Bank of Switzerland (UBS)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (16)								
13 Feb 07	0.21%	78	1.53	76.00	Mar 07	5100	97.80%	5349	-3.86%	21	398.69%	16	3,597,400	Not identified
6 Jul 07	-0.10%	72	1.15	74.40	Aug 07	5153	95.80%	5617	-2.99%	15	462.61%	17	3,787,200	Not identified
12 Oct 07	0.43%	66	0.24	67.95	Oct 07	8327	93.00%	9288	-4.44%	15	420.83%	6	818,960	30 Oct 07: announcement of losses
16 Oct 07	0.51%	68	3.06	66.50	Dec 07	35563	99.80%	10	-4.44%	13	457.52%	19	11,765,000	30 Oct 07: announcement of losses
25 Oct 07	0.06%	60	4.04	62.45	Jun 08	26487	99.40%	0	-4.65%	20	250.00%	20	12,290,000	30 Oct 07: announcement of losses
11 Dec 07	0.49%	56	1.04	56.95	Dec 07	46430	99.80%	997	-3.56%	3	299.04%	9	14,088,000	10 Dec 07: posts 10bn write-down
30 Jan 08	-0.51%	46	2.86	46.06	Mar 08	5271	81.80%	5534	-8.32%	12	261.54%	18	4'289'921	15 Feb 08: worst quarterly loss
11 Feb 08	-0.82%	36	1.51	39.74	Mar 08	3036	63.00%	3100	-8.32%	4	286.75%	20	1,609,460	15 Feb 08: worst quarterly loss
12 Feb 08	-1.03%	40	0.81	40.94	Feb 08	3374	65.80%	4654	-8.32%	3	391.36%	4	1,033,808	15 Feb 08: worst quarterly loss
22 Feb 08	-1.28%	33	0.96	35.68	Mar 08	6000	83.00%	6000	-7.42%	15	785.42%	17	2,832,300	15 Mar 08: bank run at Bear Stearn
27 Feb 08	-1.12%	24	0.37	37.48	Jun 08	15000	96.20%	15000	-7.42%	12	818.92%	14	1,785,000	15 Mar 08: bank run at Bear Stearn
04 Mar 08	-1.24%	30	1.50	32.08	Apr 08	9827	91.40%	12032	-7.42%	9	308.00%	10	1,206,100	15 Mar 08: bank run at Bear Stearn
16 May 08	-0.12%	32	1.38	32.16	Jun 08	19193	96.60%	20127	-6.95%	8	193.48%	7	5,102,400	26 May 08: more mortgage losses possible
						Call Options (13)								
27 Sep 07	-0.05%	66	0.21	62.15	Oct 2007	12495	97.40%	2647	3.04%	3	1090.50%	8	2,476,000	Not identified
23 Nov 07	-1.27%	54	1.41	49.74	Jan 08	13495	95.80%	14355	6.65%	4	258.87%	12	4,140,600	Not identified
19 Aug 09	0.89%	17	0.26	16.74	Aug 09	38269	99.80%	50240	6.40%	3	523.08%	3	4,676,400	Not identified

Table 2.14. Summary of detected informed trades for United Bank of Switzerland. For definition of entries see Page 80.

Credit Suisse Group (CS)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (16)								
13 Jul 07	-0.14%	76	2.73	82.19	Jun 08	11863	98.20%	12000	-3.44%	10	135.90%	12	581,290	Not identified
12 Oct 07	0.26%	80	2.12	75.02	Dec 07	9929	95.00%	9949	-3.73%	15	423.11%	20	1,705,300	Not identified
01 Nov 07	-0.23%	70	1.01	68.95	Dec 07	9382	94.20%	9500	-3.73%	1	754.46%	16	2,526,500	Not identified
07 Jan 08	-0.11%	60	1.97	58.84	Mar 08	9779	92.60%	11050	-8.41%	11	397.46%	11	868,470	Not identified
19 Feb 08	0.50%	52	2.46	48.72	Mar 09	13240	97.00%	14705	-6.61%	1	180.08%	20	1,150,200	20 Feb 08: new 2bn write-down
						Call Options (13)								
16 Mar 07	-0.22%	88	1.29	75.50	Apr 07	5107	95.40%	5668	3.26%	5	230.23%	6	1,175,400	Not identified
23 Nov 07	-1.01%	68	1.85	57.87	Jan 08	11118	97.00%	11475	4.72%	4	162.70%	14	3,060,800	Not identified
23 Apr 08	0.20%	51	2.23	48.31	May 08	5006	86.60%	5056	5.98%	7	240.36%	7	1,351,300	Not identified
10 Sep 08	-0.06%	54	0.74	49.92	Sep 08	9850	93.00%	10300	18.63%	8	170.27%	8	1,201,000	Not identified
18 Sep 08	-0.38%	58	1.32	45.36	Dec 08	10010	93.00%	10010	27.92%	18	346.21%	12	1,396,600	13 Oct 08: Swiss banks raise emergency

Table 2.15. Summary of detected informed trades for Credit Suisse Group. For definition of entries see Page 80.

Deutsche Bank (DBK)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (16)								
29 Jun 07	-0.23%	100	1.82	107.09	Sep 07	12660	96.60%	171	-3.55%	20	175.27%	20	4,793,800	Not identified
09 Jan 08	-0.12%	80	1.42	84.55	Feb 08	15157	94.20%	18422	-6.30%	9	507.75%	11	5,655,400	Not identified
14 May 08	0.22%	76	5.02	76.24	Jul 08	3648	71.40%	3698	-5.93%	13	180.08%	20	4,134,800	Not identified
19 May 08	0.09%	76	6.51	76.64	Dec 08	6700	89.00%	8570	-5.93%	10	133.03%	18	4,852,300	Not identified
30 May 08	-0.24%	68	1.91	68.98	Jul 08	5596	86.20%	5775	-5.93%	1	468.06%	20	3,987,000	Not identified
08 Aug 08	0.72%	58	1.06	63.07	Sep 08	5759	86.60%	7277	-5.28%	4	228.30%	10	1,131,000	Not identified
18 Sep 08	-0.56%	46	3.52	48.84	Dec 08	10425	94.60%	13405	-14.79%	17	369.60%	17	3,447,800	Not identified
19 Sep 08	-0.65%	54	2.27	58.18	Oct 08	5889	87.80%	7050	-14.79%	16	903.96%	16	10,753,000	Not identified
01 Oct 08	-0.73%	32	1.23	51.46	Dec 08	5424	85.40%	5932	-14.79%	8	796.75%	20	1,523,800	Not identified
						Call Options (3)								
30 Mar 07	0.07%	100	2.61	100.70	Apr 07	6094	91.00%	540	3.43%	14	384.29%	14	5,834,600	Not identified

Table 2.16. Summary of detected informed trades for Deutsche Bank. For definition of entries see Page 80.

Société Générale (GL)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (16)														
25 Jul 07	-0.20%	130	2.72	131.28	AUG07	3020	98.20%	0	-4.84%	12	168.75%	4	2,631,284	Not identified
9 Jan 08	-0.54%	88	1.67	94.99	FEB08	326	52.60%	0	-8.72%	9	531.14%	9	317,002	18 Jan 08: speculation of huge write-downs
16 Jan 08	-0.25%	92	3.95	94.93	MAR08	856	77.60%	0	-8.72%	4	444.05%	9	1,446,670	18 Jan 08: speculation of huge write-downs
05 May 08	0.79%	77	1.19	78.45	MAY08	520	61.10%	50	-3.16%	4	410.92%	7	187,170	14 May 08: first quarter profit drops 23.4%
16 May 08	0.18%	70	2.12	71.85	JUN08	400	48.20%	50	-3.81%	16	169.81%	10	325,128	14 May 08: first quarter profit drops 23.4%
8 Sep 08	-0.46%	58	0.85	66.81	OCT08	500	54.70%	NaN	-9.59%	6	642.35%	8	661,000	15 Sept 08: collapse of Lehman Brothers
9 Sep 08	-0.23%	67	1.42	68.50	SEP08	2285	96.20%	NaN	-9.59%	5	817.61%	7	311,230	15 Sept 08: collapse of Lehman Brothers
Call Options (8)														
28 Sep 07	0.22%	130	1.15	117.68	NOV07	3155	98.90%	550	4.11%	3	228.70%	6	2,404,519	Not identified
24 Jan 08	-1.04%	90	1.26	75.81	MAR08	3000	97.00%	0	10.63%	4	372.22%	7	5,264,540	12 Feb 08: seeks to raise $8 Billion
29 Jan 08	-1.55%	100	0.94	78.45	MAR08	1500	86.50%	120	10.63%	1	189.36%	4	4'186'500	12 Feb 08: seeks to raise $8 Billion
18 Sep 08	-0.61%	72	0.41	55.86	Oct 08	1120	71.20%	241	19.16%	2	563.41%	2	1,706,445	Not identified
28 Oct 08	-2.19%	48	0.60	33.34	Nov 08	963	60.30%	200	11.87%	6	431.67%	7	1,761,747	Not identified

Table 2.17. Summary of detected informed trades for Société Générale. For definition of entries see Page 80.

BNP Paribas (BN)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
						Put Options (16)								
14 Jan 08	-0.37%	70	1.10	73.72	FEB08	271	38.90%	0	-10.20%	6	618.18%	6	130,601	30 Jan 08: quarterly profit will slump over 40%.
15 Jan 08	-0.27%	68	0.98	72.25	Feb 08	2458	95.40%	68	-10.20%	5	546.94%	5	1,777,858	30 Jan 08: quarterly profit will slump over 40%.
16 Jan 08	-0.23%	64	2.78	73.17	Jun 08	4139	99.00%	120	-10.20%	4	172.30%	4	559,877	30 Jan 08: quarterly profit will slump over 40%.
18 Jan 08	-0.15%	64	1.68	69.39	Mar 08	1000	66.00%	20	-10.20%	2	175.60%	4	98,734	30 Jan 08: quarterly profit will slump over 40%.
26 May 08	0.02%	68	2.67	66.60	Jun 08	1000	69.60%	5	-4.08%	10	176.40%	10	659,500	Not identified
30 May 08	-0.04%	66	1.61	66.34	Jun 08	2445	94.60%	NaN	-4.08%	6	294.41%	9	1,335,947	Not identified
03 Oct 08	0.32%	70	3.08	71.35	Oct 08	141	27.60%	2	-13.47%	18	386.69%	10	156,023	28 Oct 08: crash
07 Oct 08	0.36%	66	3.03	68.43	Oct 08	511	51.80%	NaN	-13.47%	16	263.37%	8	412,633	28 Oct 08: crash
21 Oct 08	-0.83%	58	5.56	59.00	Nov 08	2500	93.00%	100	-13.47%	6	96.28%	6	3,326,778	28 Oct 08: crash
						Call Options (13)								
26 Jul 07	-0.29%	80	3.34	79.12	Sep 07	600	73.50%	0	6.08%	17	247.62%	10	1,195,800	Not identified
04 Dec 07	0.41%	70	2.50	74.00	Feb 08	1464	86.60%	0	2.77%	2	197.56%	5	954,371	Not identified
23 Sep 08	0.73%	60	1.90	64.53	Oct 08	638	62.70%	656	8.23%	9	544.83%	9	1,137,240	positive signals from US bailout program
25 Sep 08	0.70%	66	4.51	67.21	Dec 08	3045	92.50%	1000	8.23%	7	173.21%	9	8,279,315	positive signals from US bailout program

Table 2.18. Summary of detected informed trades for BNP Paribas. For definition of entries see Page 80.

HSBC (HSB)

Day	M_t	K	P_t	S_t	τ	ΔOI_t	q_t	V_t	$r_t^{\mathbf{s}}$	τ_1	$r_t^{\mathbf{o}}$	τ_2	G_t	**Event**
Put Options (12)														
12 Oct 07	0.41%	940	5.00	953.00	Oct 07	3000	93.50%	0	-2.73%	15	230.00%	6	3,450'000	Not identified
11 Dec 07	0.16%	850	12.50	851.50	Dec 07	1200	69.70%	0	-2.64%	3	112.00%	5	840,000	Not identified
07 Jan 08	0.14%	840	28.50	833.00	Feb 08	56	20.30%	0	-6.38%	11	212.28%	8	374,200	Recession hurts demand for loans
10 Jan 08	-0.18%	840	29.50	814.00	Jan 08	23	8.30%	0	-6.38%	8	193.22%	5	131,100	Recession hurts demand for loans
14 Jan 08	-0.23%	780	13.50	811.00	Feb 08	461	40.50%	106	-6.38%	6	459.26%	6	1,452,400	Recession hurts demand for loans
Call Options (5)														
10 Sep 07	-0.14%	880	18.00	870.00	Sep 07	3000	95.80%	25	2.63%	18	111.54%	8	16,500'000	Not identified
25 Sep 07	0.05%	900	12.50	909.00	Oct 07	888	66.20%	5	2.63%	7	122.22%	9	3,937,000	Not identified

Table 2.19. Summary of detected informed trades for HSBC. For definition of entries see Page 80.

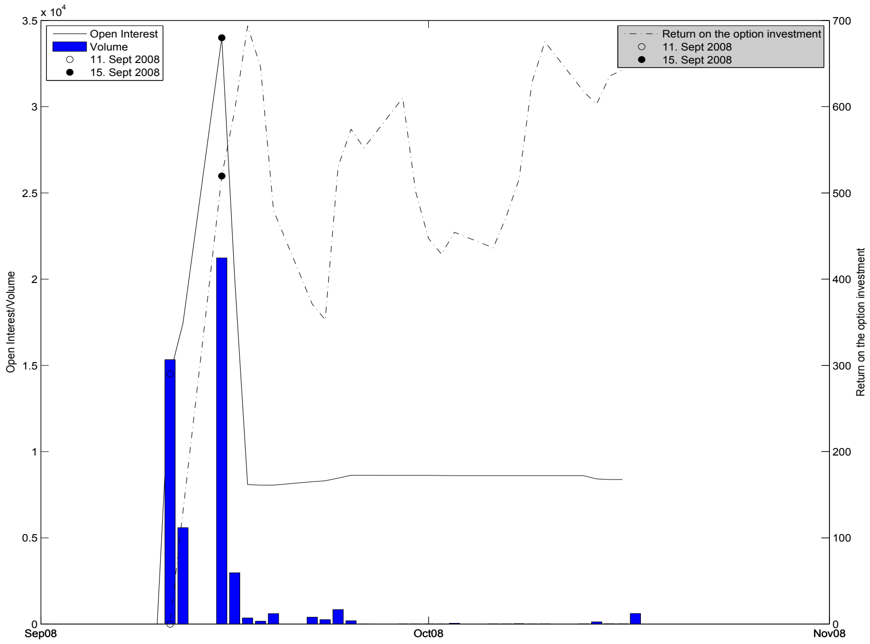

Figure 2.1. Selected put option with underlying stock AIG before the Federal Reserve Board announced that it would take a nearly 80% equity stake in AIG - effectively taking over the firm - and would provide an $85 billion loan on September 15, 2008. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line, the option return (right y-axis). The empty circle is the day of the transaction, the filled circle is the announcement day, September 15, 2008. This put option had a strike of $8 and matured at the end of October 2008.

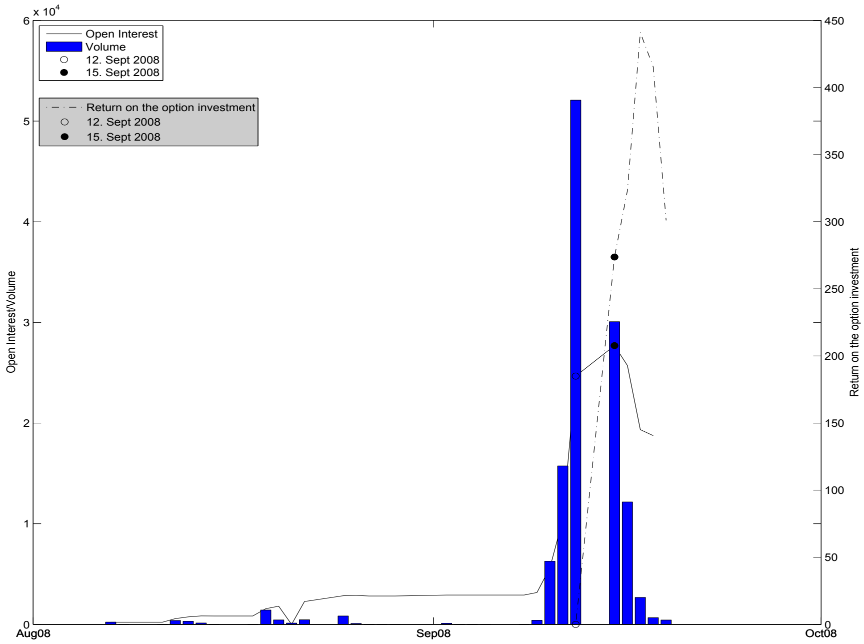

Figure 2.2. Selected put option with underlying stock AIG before the Federal Reserve Board announced that it would take a nearly 80% equity stake in AIG - effectively taking over the firm - and would provide an \$85 billion loan on September 15, 2008. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line, the option return (right y-axis). The empty circle is the day of the transaction, the filled circle is the announcement day, September 15, 2008. This put option had a strike of \$10 and matured at the end of September 2008.

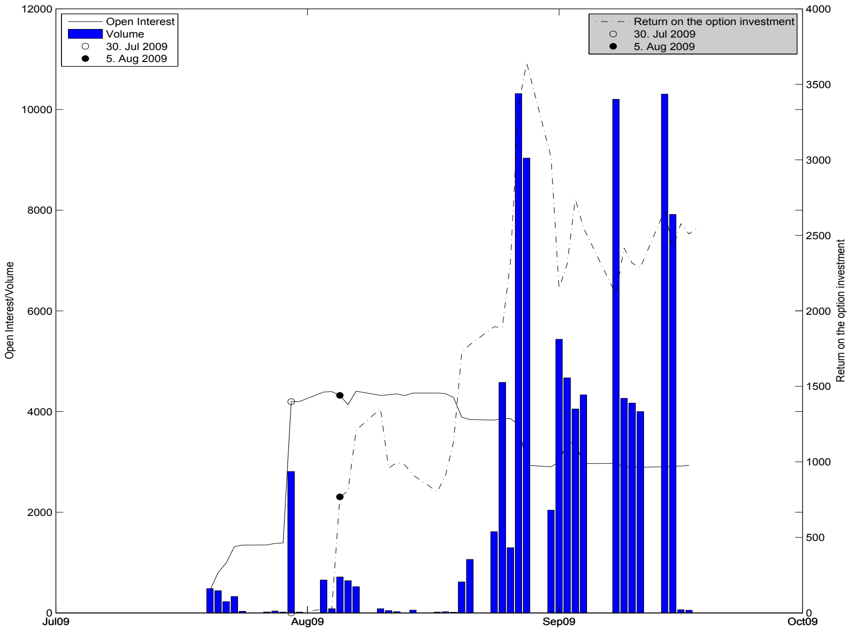

Figure 2.3. Selected call option with underlying stock AIG before the August 7, 2009 quarterly profit announcement that almost doubled the stock value. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line, the option return (right y-axis). The empty circle is the day of the transaction, the filled circle is the announcement day, August 7, 2009. This call option had a strike of $15 and matured at the end of September 2009.

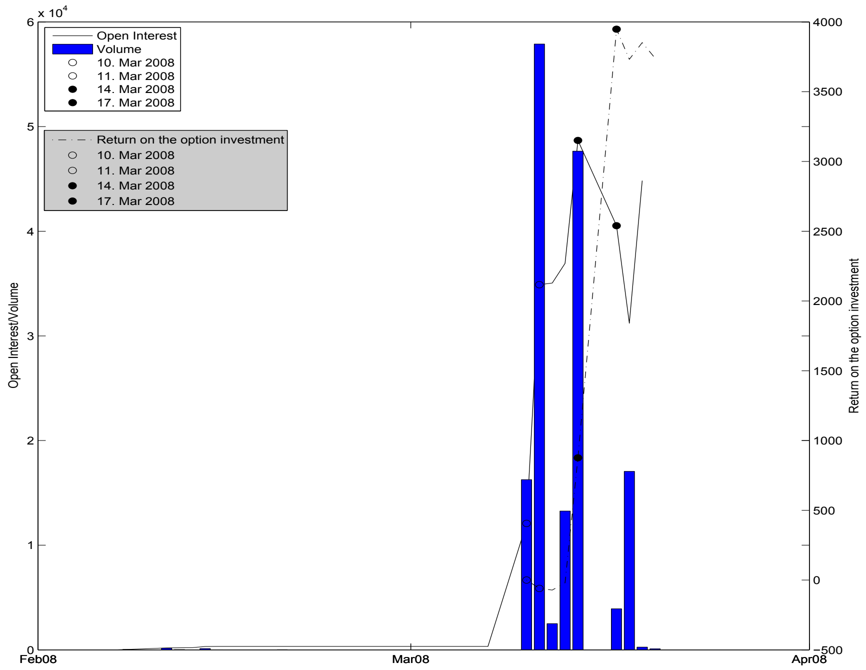

Figure 2.4. Selected put option with underlying stock BSC before the collapse of Bear Stearns on March 17, 2008. The option trade takes place on March 10, 2008. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line, the option return (right y-axis). The empty circle is the day of the transaction, the filled circle is Monday, March 17, the day Bear Stearns shares dropped nearly 90 % to $2.86. This put option had a strike of $30 and matured at the end of March 2008.

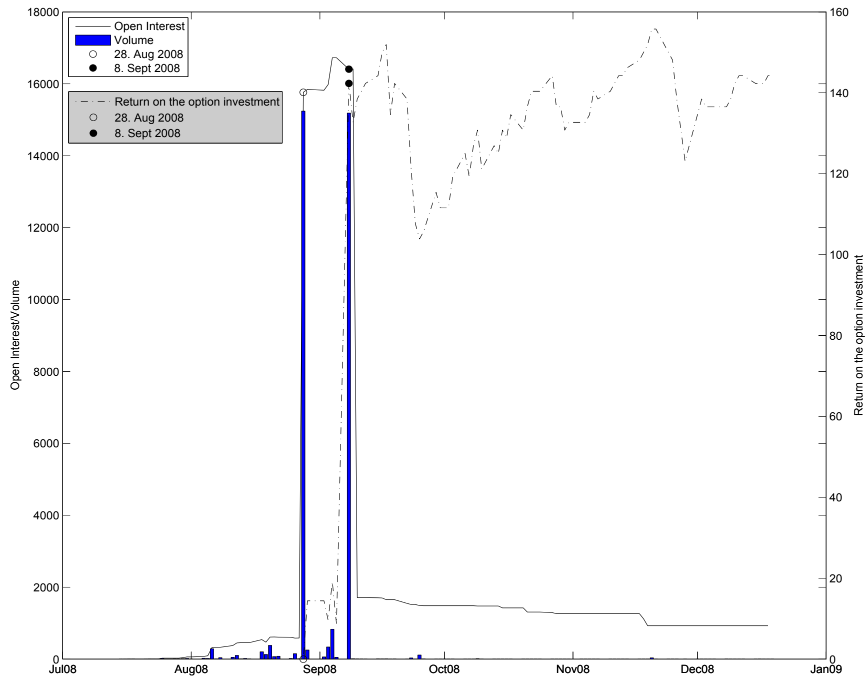

Figure 2.5. Selected put option with underlying stock FNM before the federal takeover on September 5, 2008. The option trade takes place on August 28, 2008. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line the option return (right y-axis). The empty circle is the day of the transaction, August 28, 2008, and the filled circle is Monday, September 8, when the stock price of Fannie Mae crashed by almost 90% to under $1. This put option had a strike of $7 and a time-to-maturity of more than 100 days.

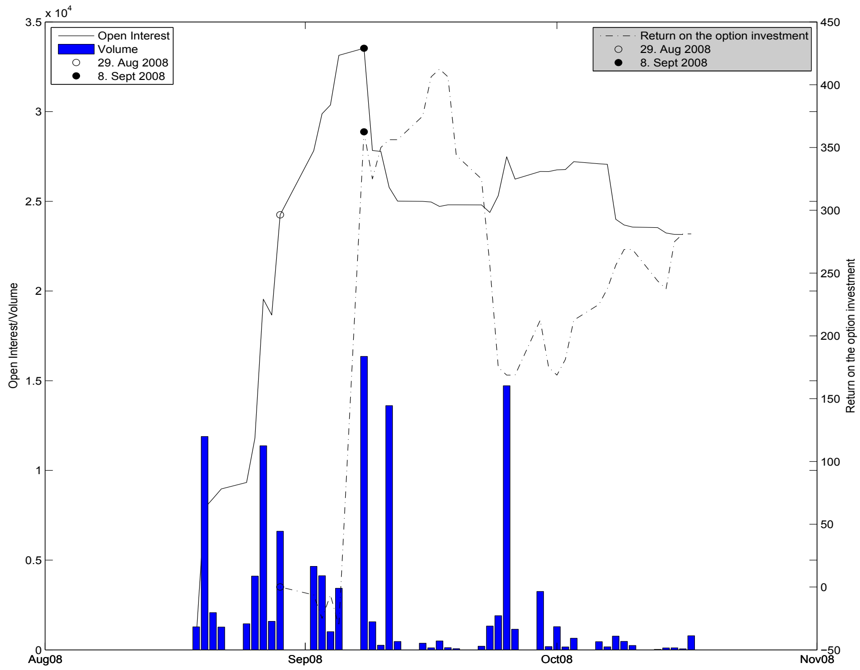

Figure 2.6. Selected put option with underlying stock FNM before the federal takeover on September 5, 2008. The option trade takes place on August 29, 2008. The solid line shows the daily dynamic of open interest, the bar shows the corresponding trading volume (left y-axis) and the dash-dot line the option return (right y-axis). The empty circle is the day of the transaction, August 29, 2008, and the filled circle is Monday, September 8, when the stock price of Fannie Mae crashed by almost 90% to under $1. This put option had a strike of $3 and a maturity at the end of October 2008.

References

[1] Marc Chesney, Remo Crameri, and Loriano Mancini. Detecting informed trading activities in the options markets. *Working paper*, 2009. University of Zurich.

3

Informational Content of the Daily Imbalance Between Put and Call Options

Remo Crameri

Summary. We study the informational content of open interest when the linkage between option market variables and subsequent price movements in the underlying stock are investigated. A daily statistic which measures the imbalance between new issued puts and calls is defined. Conditional on it we estimate non-parametrically the whole cumulative distribution of several indicators of future market activities. Differences between the unconditional and conditional distribution are used as a measure for the predictive power of our statistic. We empirically show that whenever the imbalance between new issued puts and calls takes extreme values, the conditional distribution exhibits significant changes with respect to its unconditional counterpart: when the number of new issued put options is large compared to the number of new call options, the conditional distribution functions of the idiosyncratic return noise process becomes heavier on the left side and a large drop is more likely to follow. In the opposite scenario, when the statistic exhibits a large imbalance in favor of call options, the idiosyncratic return noise tends to be higher than after calm days. Our findings confirm the informational content of large daily changes in open interest.

Keywords: Options, Information Content, Open Interest.

JEL Classification: G12, G13, G14, G17, G34, C61, C65

3.1 Introduction

Option contracts have been widely accepted as one of the most useful derivative securities of the last decades. As in the stock market, daily trading volume, bid-ask quotes, prices and volatility are considered to be the driving components in the analysis of trading activity in the options market. However, unlike in the stock market in which there is a fixed number of shares to be traded, option trading can involve the creation of new positions whenever a new party agrees to underwrite a new contract and another one is disposed to take the opposite position. Therefore the number of existing (not yet exercised) option contracts can be quite volatile and may significantly change from one day to the other. This fact is confirmed by historical daily data from the CBOE (Chicago Board Options Exchange). The technical term used for the total number of option contracts for a specific underlying asset, strike price and expiration date that are currently open at the end of a trading day is *Open Interest*. This concept is widely used among options traders and is one of the data fields on every option quote display. Together with daily trading volume, the dynamic (daily changes) of open interest might be considered as an indicator of different activities in the options market and might have predictive power on the future development of the underlying's price. For example, the liquidity of an option can be described by its corresponding open interest, and trading in an (existent) option can be considered unusually and exceptionally high whenever the daily volume far exceeds the existing open interest on a given day. In addition, daily changes in open interest may contain/reveal what other investors think/know about the future value of the underlying stock. Finally, daily increments of open interest can help in determining how much new money is flowing into or out of a market.

Daily changes in open interest play the main role in this paper. As already noted in several papers, option market anomalies (in terms of unusually high volumes or open interest) are an important indicator of new information flowing into the market. [19] compare for example stock and option market activities and first observe that there are abnormal activities in both markets before the M&A announcement day. These activities turn out to be more intense in the options market. They find a dramatic and significant increase in both variables, open interest and volume, and point out that using open interest as an indicator has two decisive advantages: first of all, it is much less volatile than volumes and, secondly, daily volume can be affected by short-term speculation (a trader opens a position in the morning and closes it in the evening). Therefore the (total) daily increment in open interest corresponds to the number of new option contracts issued on the market at that specific day. The dynamic of such a variable might provide new insight into the information process on several levels which is not available when analyzing the stock market.

It is important to analyze the informational content of unusual activities in the options market as it enhances our understanding of how (or if) new information is possibly reflected in asset prices. The analysis presented in this paper investigates therefore the linkage between option market variables and subsequent price movements of the underlying stock.

In this paper we study the informational content of daily changes in open interest. First we define a daily statistic which represents the imbalance between new issued put and call options. Second, we compute three different measures of future market activities: the mean, the maximum and the minimum of the idiosyncratic noise of the return process over a time period of 5 trading days. We then analyze the predictive power of our statistic on these three measures by comparing the conditional distribution functions with their unconditional counterparts. Our findings suggest that the predictive power of our statistic is appreciable: when conditioning on high/low levels of imbalance, the distribution functions of all three measures show significant changes compared to the unconditional ones. These differences turn out to be more pronounced when looking at the minimum and maximum idiosyncratic noise: when the number of new issued put options is large compared to the number of new calls, the conditional distribution function becomes heavier on the left side and a large drop in the idiosyncratic return noise is more likely to follow. In the opposite scenario, when the statistic exhibits a large imbalance in favor of call options, the idiosyncratic return noise tends to be higher than after calm days.

Our paper is related to existing literature in several ways. We share the idea with a number of previous studies that option markets are attractive to informed traders. [11] and [8] focus on option trading volumes. In the first article, their model predicts an information role for the volume of specific types of option trades. Empirical analysis of the model confirms the predictions. The second paper provides evidence that due to non-public information possessed by options traders, transaction volume in options markets generates information concerning future stock prices. [21] present strong evidence that options trading volume and open interest contain information about future stock prices. [18] provide detailed descriptive statistics on purchased and written open interest and on open buy and sell volumes of several classes of investors. [22] analyzes option market activities around the terrorist attacks of September 11 and concludes that several anomalies are observable. Most of these papers use unique databases with publicly and non-publicly available information. Their analysis takes place in a standard linear regression framework. [9] develop a procedure for the detection of informed trading activities in the options market. This paper has important similarities to [22] and [9], such as using open interests to investigate the impact and the transmission of information from the options market to the underlying asset market and vice versa. As in [22] we perform unconditional and conditional analysis. There are also important differences from [22] concerning the data, methodology and aims. [22] takes advantage of a rich database where option trades are separated in long and short positions and classified according to the agents who initiated them. Such a database allows for an in-depth analysis, but is, unfortunately, not easily accessible to the public. We use publicly available data such as total daily open interest and stock returns so that our methodology can be readily applied elsewhere. The data used in this paper partially corresponds to the data analyzed in [9]. From a methodological point of view, there are however important differences: in this paper we simultaneously analyze call and put options, whereas [9] study transactions on the options market which are characterized, among others, by an unusually high increment in put open interest exclusively. Call options are not considered. We construct a (daily) statistic which measures the imbalance between new issued puts and calls, and, conditional on this, we estimate non-parametrically the whole cumulative distribution of several indicators of future market activities. The difference between unconditional and conditional distribution is used as a measure of the predictive power of our statistic. Obviously, the use of high frequency data would permit a more detailed analysis. From a conceptual point of view, our paper is related to [13] which tests the informational content of trading in option strategies. In both papers, the predictive power of options trading is analyzed using daily statistics. [13] examine the information content of the daily order flow in option strategies by taking the difference between the total number of contracts of buyer-initiated trades and the total number of contracts of seller-initiated trades on a given day. They construct separate daily order flow measures for directional strategies, which include all option strategies with large delta and small vega exposure, and volatility strategies, which include all option strategies with small delta and large vega exposure. Evidence that the order flow in volatility strategies predicts next day realized volatility (consistent with traders having some non-public information about volatility) is reported. No corresponding evidence of directional strategies and the next day's index return is found. The classification of buyer- and seller-initiated transactions is of crucial importance in [13] and is only possible due to the availability of options high frequency data. In our paper, we find empirical evidence of our daily statistic even when considering a less rich data base.

The paper is organized as follows: in Section 3.2 we introduce our open interest statistic. Section 3.3 describes the database. Section 3.4 shows our empirical results. Section 3.5 concludes.

3.2 Open interest statistic

This section introduces a new open interest statistic that we use to analyze the linkages between option and stock markets. Let OI_t denote the sum of open interest at the end of day t of all available option contracts for a given company. Clearly when $OI_{t+1} > OI_t$ the number of outstanding options increases. However, if for instance the number of outstanding puts increases, it can be due to an increase in the demand for protective puts, hedging possible drops in stock prices or due to liquidity (noise) trading. To disentangle the two types of trading a quite sophisticated database would be necessary.[1] In this paper we try to achieve this challenging goal by using publicly available data. Hence all our results should be interpreted in a conservative way. It is reasonable to expect that by using a richer database more stringent results could be achieved. Unfortunately, these data are not easily available to the public.

Our basic idea is to measure the imbalance between the increments in put and call open interests. Assuming that hedging and noise trading equally involve puts and calls, when we observe a relatively large increment in put options compared to the change in call option positions, we can expect that the trading in put options is mainly undertaken by informed agents. The impact on the stock market is expected therefore to be more

[1] Model based approaches could be used to classify the transactions; see for instance [11]. However, these approaches can only provide an approximate classification of trades.

pronounced and a fall in the underlying asset is likely to occur in the near future. Our open interest statistic, I_t, is as follows

$$I_t := \frac{OI_{P,t} - OI_{P,t-1}}{OI_{P,t-1}} - \frac{OI_{C,t} - OI_{C,t-1}}{OI_{C,t-1}}, \tag{3.1}$$

where $OI_{P,t}$ and $OI_{C,t}$ denote the sum among different strikes and maturities of put and call open interest on day t for the company we are interested in. We will compute statistic I_t for 20 companies with options traded on the CBOE. When we need to emphasize the dependence of I_t on the specific stock i we use the notation $I_{i,t}$. The difference $OI_{P,t} - OI_{P,t-1}$ is the increase in the number of outstanding put option contracts, and similarly $OI_{C,t} - OI_{C,t-1}$ for call options. The interpretation of I_t is as follows. When I_t is sufficiently large we expect the stock price i to fall and vice versa. When informed traders act on privileged information, the corresponding large positions are reflected in a strong increase or decrease in I_t. By construction, we focus therefore only on abnormal trades that increase open interest. Whether or not a large demand for put or call options is able to increase the corresponding open interest is dependent on the dealers' inventories. If a large buy order comes in at a time when the dealer holds a large positive inventory, then the corresponding open interest will not change. Thus, our approach must be interpreted in a conservative way: a large demand for put or call options that can be satisfied without the creation of new positions is therefore neutralized by our statistic. The I_t are also affected by the baseline activity in the options market which might not equally impact put and call open interests. We try to filter out this effect using a regression model. The impossibility of accounting exactly for this phenomenon reduces the predictability of our method. Hence, also in this respect, our results should be interpreted in a conservative manner.

Our statistic I_t has three main advantages over the put-call volume statistic widely used in the financial press and discussed in [22]. Firstly, I_t is constructed using publicly available data and hence it can be readily computed for any option of interest. Secondly, as in [22] we consider increments in open interests $OI_t - OI_{t-1}$, but, as he points out, such a difference is likely to depend on the number of option contracts associated with a given underlying asset. The ratios in equation (3.1) account for this dependence and allows us to meaningfully compare the different statistics I_t across different stocks and periods. Finally, taking the difference as opposed to the ratio between relative increment of put and call open interest avoids explosive behaviors of statistic I_t when the increment in call open interest is approximately zero. This phenomenon occurs with less liquid options. Controlling the range of variation of I_t is important if we wish to achieve meaningful statistical results when I_t is regressed on explanatory variables. For this reason we transform the statistic I_t as follows:

$$\mathcal{I}_t := \Phi\Big(\frac{I_t - mean(I_t)}{std(I_t)}\Big), \tag{3.2}$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution. By construction, $\mathcal{I}_t$ falls into the unit interval. In the case in which the imbalance between new issued put and call options is around zero, the statistic $\mathcal{I}_t$ will be near to 0.5. The first open interest statistic investigated by [22] is related to the put-call ratio. However, over the last decades daily open interest has dramatically increased. Hence statistics based on absolute differences of open interests such as the above-mentioned statistic are more noisy than statistics based on relative differences as we propose. This is the main reason for which we use $\mathcal{I}_t$ in (3.2) instead of the first Poteshman statistic. The upper graph in Figure 3.1 gives an overview of the statistic $\mathcal{I}_t$ for the underlying stock American Airlines (AMR). By construction, the time series $\mathcal{I}_t$ oscillates around 0.5. The days in which $\mathcal{I}_t$ is high indicate that the relative increment in put options was larger than the one in call options. In the same figure we marked five different days. These correspond to the days detected in [9] in which potential informed trading activities using put options took place. The high value of our statistic $\mathcal{I}_t$ confirms the strong asymmetry in the demand for new put and call options during these days and supports the choice of our statistic $\mathcal{I}_t$.

3.2.1 Predictive power of open interest statistic on return innovation

It is reasonable to expect that both $\mathcal{I}_t$ and stock return r_t are affected by a number of factors. From a rational point of view, the imbalance between put and call open interests $\mathcal{I}_t$ can be affected by past stock returns as a consequence of hedging strategies. In a bear market for example, a large hedging demand for protective puts will dominate the demand for call options. This might therefore increase the value of statistic $\mathcal{I}_t$. In the opposite case, a bullish trend in the stock market may possibly decrease statistic $\mathcal{I}_t$ as a consequence of high demand for call options. For comparative purposes, during times of high volatility, uncertainty of the directional movements of the underlying assets or disagreement among traders on future price movements, statistic $\mathcal{I}_t$ is likely to take a neutral value around 0.5. In order to take into account possible factors which might have an impact on the behavior of statistic $\mathcal{I}_t$, we concentrate on the filtered statistic η_t computed as

$$\mathcal{I}_t = a_0 + \sum_{j=1}^{n}(b_j\, r_{t-j} + c_j\, r_{t-j}^2 + d_j\, \mathcal{I}_{t-j}) + \eta_t. \tag{3.3}$$

By construction, the innovations η_t represent the (transformed) imbalance between new issued put and call options at day t which cannot be attributed to stock returns or volatility trends (approximated by the squared returns) of the last n trading days. A graphical representation of η_t is given in the second graph in Figure 3.1. Aside from the down shift, the behavior is qualitatively the same as in the first graph. This is confirmed by the regression coefficients reported in Table 3.1. The estimated constant term $\hat{a}_0$ in equation (3.3) is around 0.5 and in all cases statistically significant. In the majority of cases, the coefficients related to the one lag past returns ($\hat{b}_1$) is negative and significant at the 5% level. This could be an indication that in periods of bearish/bullish times, investors tend to augment their demand for protective puts/calls with a one-day delay. Consequently, a day with a large positive/negative return decreases/increases the level of the statistic $\mathcal{I}_t$ in the following day. The coefficient of the two-day lagged return ($\hat{b}_2$) does not seem to have a significant impact on the statistic: it is significant in some cases but with a changing sign. With respect to past squared returns, statistic $\mathcal{I}_t$ does not react uniformly across the analyzed companies. The same conclusion holds for the lagged statistic values. Overall, the null hypothesis that the R^2-statistic of the regression model (3.3) is equal to zero is rejected due to the high $F-$test values reported in Table 3.1.

According to standard CAPM-type models stock returns are driven by systematic and unsystematic shocks, among others. Moreover, a substantial amount of empirical evidence suggests that daily stock return volatility is stochastic and mean reverting. It also responds asymmetrically to positive and negative returns (see for instance [14]). Stochastic volatility is often modeled using extensions of the autoregressive conditional heteroscedasticity (ARCH) model proposed by [12].[2] To model stock returns we use an asymmetric GARCH specification with an empirical innovation density. The GARCH model of [3] accounts for stochastic, mean reverting volatility dynamics. The asymmetry term is based on [15] (GJR). The empirical innovation density captures potential non-normalities in the true innovation density.

[2] Comprehensive surveys of ARCH and related models are [4] and [5]. In a continuous time setting, stochastic volatility diffusion models are commonly used; surveys of this literature are for instance [14] and [23].

Let $r_t = S_t/S_{t-1} - 1$ be the daily return of the stock we are analyzing with price S_t at time t, then the asymmetric GJR GARCH model is

$$\begin{aligned} r_t &= a + b\, r_{M,t} + \varepsilon_t, \\ \sigma_t^2 &= \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma J_{t-1}\varepsilon_{t-1}^2, \end{aligned} \tag{3.4}$$

where $r_{M,t}$ is the market return (thereafter approximated by the S&P 500), $\varepsilon_t = \sigma_t\, z_t$, $z_t \sim f(0,1)$ and $J_{t-1} = 1$, when $\varepsilon_{t-1} < 0$ and $J_{t-1} = 0$, otherwise. When $\gamma > 0$ the model accounts for the leverage effect,[3] that is, bad news ($\varepsilon_{t-1} < 0$) raises future volatility more than good news ($\varepsilon_{t-1} \geq 0$) of the same absolute magnitude. We estimate the model parameters using maximum likelihood. Under quite general conditions [6] show that this technique provides consistent parameter estimates even when the true innovation density f is non-normal. In the last case the parameter estimates are pseudo maximum likelihood (PML) estimates. The scaled return innovations z_t is obtained by dividing each estimated return innovations $\hat{\varepsilon}_t$ by its estimated conditional volatility $\hat{\sigma}_t$. z_t is an estimate of the idiosyncratic noise of the underlying stock. Hence, market changes do not explain the *sign* of the shock z_t, and changing stock volatility does not affect the *size* of the shock. Accounting for this phenomenon allows us to disentangle those variations in stock returns due simply to market conditions from those which are unexplained by them (the innovations z_t). This constitutes a major difference between our approach and those in previous literature. Figure 3.2 and Table 3.2 summarize the result of the GARCH model presented in equation (3.4). The two graphs in Figure 3.2 represent the effect of extracting the idiosyncratic noise z_t from the row return process r_t. Some extreme returns in the first graph are not in the second one, suggesting that on these days either market changes or high volatility were the main cause for such an extreme return. Later we test the predictive power of high/low values of the filtered statistic η_t on future values of z_t. Table 3.2 displays the estimated coefficients of the asymmetric GARCH model (3.4): as expected, the coefficient $\hat{a}$ does not statistically differ from zero, whereas $\hat{\beta}$ is significant for all companies and takes values between 0.4 and 1.7. The coefficient $\hat{\gamma}$ related to the leverage effect in (3.4) is positive for almost all companies.

[3] [2] introduced the name leverage effect and suggested that a large negative return increases the financial and operating leverage, and raises equity return volatility; see also [10]. [7] suggested an alternative explanation based on market risk premium and volatility feedback effects; see also the more recent discussion by [1]. We shall use the name leverage effect as it is commonly used by researchers when referring to the asymmetric reaction of volatility to positive and negative return innovations.

3.2.2 Predictability of filtered statistic

In this section we empirically test the predictability of the (filtered) statistic η_t on the future return innovations z_t. The main interest is focused on the differences between the conditional distribution and its unconditional counterpart. We test the predictive power at several levels when considering future movements of the idiosyncratic noise z_t. More precisely, for every company in our sample, we first measure the future idiosyncratic activities of the return process during a time-window of n days by computing indicators $\bar{z}_t$ such as

$$\bar{z}_t := \frac{1}{n}\sum_{i=1}^{n} z_{t+i}, \bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}, \bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}. \tag{3.5}$$

We now test the impact of the statistic innovations η_t on the future return innovations z_t by comparing $F^{\text{uncond}}(\bar{z}_t) := \mathbb{P}[\bar{Z}_t \leq \bar{z}_t]$, the unconditional distribution of $\bar{z}_t$, with $F^{\text{cond}}(\bar{z}_t|\eta_t) := \mathbb{P}[\bar{Z}_t \leq \bar{z}_t|\eta_t]$, the conditional distribution of $\bar{z}_t$, when conditioning on the filtered statistic η_t. We estimate unconditional probability distributions using standard quantile regression methods as used in [17], whereas we estimate conditional probability distributions non-parametrically using the local polynomial regression model of the adjusted Nadaraya Watson estimator and the bootstrap method proposed by [16]. This last procedure, which is less common than quantile regression, is recalled in the appendix.

Define furthermore

$$\mathcal{T}_\alpha^{\text{high}} := \{t; \eta_t > q_\alpha^\eta\}\ ,\ \mathcal{T}_\alpha^{\text{low}} := \{t; \eta_t < q_\alpha^\eta\}, \tag{3.6}$$

with q_α^η being the α-quantile of the filtered statistic innovation η_t. Intuitively, for high values of α, say 90%, the set $\mathcal{T}_\alpha^{\text{high}}$ contains all days on which the (filtered) imbalance between new issued puts and calls was unusually high. In the opposite case, when α is low, say 10%, days on which the number of new calls massively prevails over the number of new puts belongs to the set $\mathcal{T}_\alpha^{\text{low}}$.

In order to get a measure of how the distribution of $\bar{z}_t$ changes when moving from the unconditional to the conditional setting, we proceed as follows: we define the unconditional distribution $F^{\text{uncond}}(\bar{z}_t)$ as being the benchmark distribution. Next, we divide its domain into four regions:

$$R_1 := (-\infty, q_{0.40}^{F^{\text{uncond}}}], R_2 := (q_{0.40}^{F^{\text{uncond}}}, q_{0.50}^{F^{\text{uncond}}}], \tag{3.7}$$

$$R_3 := (q_{0.50}^{F^{\text{uncond}}}, q_{0.60}^{F^{\text{uncond}}}], R_4 := (q_{0.60}^{F^{\text{uncond}}}, \infty), \tag{3.8}$$

where $q_\alpha^{F^{\text{uncond}}}$ denotes the α-quantile of the unconditional distributions. For $F^{\text{cond}}(\bar{z}_t|\eta_t)$ we compute now the probability mass falling into the regions R_i, $i = 1, ..., 4$. By construction, 40%, 10%, 10% resp. 40% of $F^{\text{uncond}}(\bar{z}_t)$ will fall into R_1, R_2, R_3, resp. R_4. Intuitively, if the statistic $\mathcal{I}_t$ has predictive power on future stock returns, R_1 should contain more probability mass for days in $\mathcal{T}_\alpha^{\text{high}}$ and less for days in $\mathcal{T}_\alpha^{\text{low}}$. For the region R_4 the opposite should hold. In the next section we discuss our results.

3.2.2.1 Test on the causality direction

Before testing the predictability of our statistic η_t on the future values of z_t, we check whether the causality direction we are conjecturing is correct: it might be the case that predictive power is induced by reverse causality, that is past returns innovations z_t, might explain future behavior of the filtered statistic η_t. To investigate this point we proceed as follows. First we filter η_t using an AR(5) model

$$\eta_t = a_0 + \sum_{j=1}^{5} b_j\, \eta_{t-j} + \epsilon_t$$

and then we regress the filtered statistic ϵ_t on past returns innovations

$$\epsilon_t = \alpha_0 + \sum_{j=1}^{5} (\beta_j\, z_{t-j} + \gamma_j\, z_{t-j}^2) + \mu_t.$$

The first regression is necessary to remove possible autocorrelations in η_t which could bias the results. The second regression investigates whether past returns innovations influence the future development of η_t. This procedure is repeated for each stock in our database. The corresponding t-statistics and F-test for the null hypothesis of insignificant regression coefficients are reported in Table 3.3. Overall past stock returns innovations do not appear to explain future values of the filtered open interest statistic. These findings confirm the causality direction we are interested in[4].

3.3 Data

Options data are from the Chicago Board Options Exchange (CBOE) and provided by OptionMetrics. Stock prices are downloaded as well from OptionMetrics to avoid non-synchronicity issues between options and stock prices. Stock splits and spin-offs are eliminated using information from the CRSP database. We eliminate obvious data errors such

[4] Note that this can be traced back to the different filtering procedures as well.

as open interest reported at zero for all existing options by excluding these days from our analysis. The companies analyzed belong to several industrial sectors. In alphabetical order these are: American Airlines (AMR), AT&T (ATT), AXA (AXA), Boeing (BA), Bank of America (BAC), Citigroup (C), Delta Air Lines (DAL), Hewlett Packard (HPQ), J.P. Morgan (JPM), KLM (KLM), Coca-Cola (KO), Lockheed Martin (LMT), Merrill Lynch (MER), Marsh & McLennan Companies(MMC), Philip Morris (MO), Monsanto (MON), Morgan Stanley (MWD), Rockwell Automation, Inc. (ROK), United Airlines (UAL) and United Technologies Corp. (UTX). Most sample data range from January 1996 to April 2006. Options data for DAL and KLM are available only for somehow shorter periods. In equation (3.4), the $S\&P500$ is taken as an approximation for the market.

3.4 Empirical results

3.4.1 Predictability on the mean of the return innovation

We investigate whether or not our filtered statistic η_t conveys information on the future unsystematic shocks of returns. Before applying the procedure described in the previous section, we test the predictive power of the statistic η_t on $\bar{z}_t := \frac{1}{n}\sum_{i=1}^{n} z_{t+1}$ as a preliminary result, by looking at its conditional mean for days belonging to $\mathcal{T}_\alpha^{\text{high}}$ and $\mathcal{T}_\alpha^{\text{low}}$. Intuitively, if the statistic η_t conveys information about future movements of the idiosyncratic return z_t, we expect the mean of the time-series $\bar{z}_{\tilde{t}}$ to be negative for $\tilde{t} \in \mathcal{T}_\alpha^{\text{high}}$ and α high, and positive when $\tilde{t} \in \mathcal{T}_\alpha^{\text{low}}$ for low values of α. Formally, the null hypothesis that filtered η_t has no impact on the filtered stock returns z_t can be formalized as

$$H_0 : E[\bar{z}_t|\eta_t > q_\alpha^\eta] = 0, \tag{3.9}$$

where q_α^η is α-quantile of η_t. In our empirical application we set $n = 5$, corresponding to a trading week. Under the GARCH model (3.4), the innovations z_t are approximately iid, and we can then test the null hypothesis (3.9) by simply regressing the conditional innovations, $\bar{z}_t|\eta_t > q_\alpha^\eta$, on a constant, and testing whether the constant is significantly away from zero. We also study whether low values of the filtered statistic η_t have an impact on future idiosyncratic shocks, testing $H_0 : E[\bar{z}_t|\eta_t < q_\alpha^\eta] = 0$.

Table 3.4 shows the conditional average of the innovations and the p-values of the regressions for each stock in our database when conditioning on η_t takes place. With respect to the unconditional mean, all values are statistically non-different from zero (high p-values). The conditional means tend to exhibit a different behavior: after days belonging to $\mathcal{T}_\alpha^{\text{high}}$, the mean of the innovations computed over a window of five trading days tends to be negative and vice versa when we look at days in $\mathcal{T}_\alpha^{\text{low}}$. Hence our statistic seems to have predictive power for the expected value of future idiosyncratic shocks of most companies. As a robustness check we repeat the previous analyses testing the null hypothesis (3.9) for different values of n. We obtain similar results which are not reported here, but available from the authors upon request. All additional analyses confirm that the previous results are quite robust with respect to the choice of the filtering procedure for η_t and the length of the time window n.

3.4.2 Predictability on the whole distribution of the return innovation

In the previous subsection, the predictability of the (filtered) open interest statistic η_t on the mean of z_t has been tested. In the following section we analyze the impact of the statistic on the whole distribution function of z_t. We are therefore interested in the impact of η_t not only on the mean, the location parameter, but especially on the shape and scale properties of the conditional distribution. In particular, the main interest is on the behavior of the conditional distribution with respect to the unconditional one, and how key characteristics of the involved distribution functions change when conditioning on the filtered statistic η_t. For the three measures of activities in the idiosyncratic process z_t defined in equation (3.5), we compute the unconditional distribution $F^{\text{uncond}}(\bar{z}_t)$ via quantile regression and its conditional distributions $F^{\text{cond}}(\bar{z}_t)$ using the non-parametric method of local polynomial regression model and the bootstrap method proposed by Hall, Wolff, and Yao (1999). We use the bootstrap procedure for the choice of the optimal bandwidth. In quantile regression, one needs to start with an arbitrary grid of points q_k in the unit interval (having the meaning of the quantiles $q_k := F(\bar{z}_t^k)$), and compute the corresponding $\bar{z}_t^k$, $k = 1, ..., K$. We choose equidistant points q_k, such that $q_1 = 0.001$, $q_{1000} = 1$ and $\Delta q := q_k - q_{k-1} = 0.001$. The non-parametric method of local polynomial regression model computes the corresponding quantiles q_k, $k = 1, ..., K$ for a given grid of points $\bar{z}_t^k$. For technical reasons, we truncate the domain of attainable values for $\bar{z}_t^k$ even though this would be the whole real line. Clearly, the graphical representation of the cumulative distribution function is given by the plot $(\bar{z}^k, q_k)_{k=1,...,K}$. Our main results are described by several statistical quantities of the unconditional and conditional distributions. For the unconditional and conditional distributions, we report the percentile values in steps of 10%, the probability mass falling into the regions R_1, ..., R_4 and key values such as the mean, variance, skewness and kurtosis. Approximations for the latter are computed by substituting integrals through Riemann sums. Generally, for a given function G, we have

$$E[G(\bar{Z})] := \int_{-\infty}^{\infty} G(\bar{z})dF(\bar{z}) = \int_{0}^{1} G(F^{-1}(q))dq \approx \sum_{k=1}^{K} G(\bar{z}^k)(q_{k+1} - q_k). \tag{3.10}$$

For the mean estimation of $\hat{\mu}$, one needs to choose $G(x) := x$, for the variance $\hat{\sigma}^2$ $G(x) := (x - \hat{\mu})^2$, for the skewness $G(x) := (x - \hat{\mu})^3/\hat{\sigma}^3$ and for the kurtosis $G(x) := (x - \hat{\mu})^4/\hat{\sigma}^4$. We can easily do the calculations in equation (3.10) once we have computed the series of estimated points $(\bar{z}_t^k, q_k)_{k=1,...,K}$, as previously described.

Tables 3.5 and 3.7 report our results when using the three measures of activities in the idiosyncratic process z_t defined in equation (3.5). We show the complete and detailed results for a small number of companies only. Tables for the missing companies are available upon request from the authors and do not significantly differ from the ones shown here. In order to efficiently present and summarize our findings, we compute in Tables 3.6 and 3.8 the average difference across companies between the statistical quantities (percentiles, probability mass falling into the regions R_1, ..., R_4 and key values as the mean, variance, skewness and kurtosis) when moving from the unconditional to the conditional distribution.

3.4.3 Predictive power of the statistic η_t on $\bar{z}_t := \frac{1}{n}\sum_{i=1}^{n} z_{t+i}$

Following the procedure described in the previous section, we report the result of our analysis when looking at the impact of η_t on the first measure defined in equation (3.5), $\bar{z}_t := \frac{1}{n}\sum_{i=1}^{n} z_{t+i}$. Table 3.5 displays the results for companies AMR, ATT, DAL, LMT and MWD. By conditioning on four different levels of η_t, we estimated the whole distribution function non-parametrically using the local polynomial regression method. The first four companies are quite representative of our conjecture with respect to the impact of the statistic η_t on the future values of the idiosyncratic shocks z_t. For some of the companies analyzed the results are less indicative, as in the case of MWD. For the majority of the companies analyzed, lower percentiles of the conditional distribution functions tend to be smaller than their corresponding unconditional counterparts after days belonging to $\mathcal{T}_\alpha^{\text{high}}$. Skewness and mean are lower as well, meaning that high values of η_t tend to be followed on average by low values of the idiosyncratic noise z_t. After days in $\mathcal{T}_\alpha^{\text{low}}$, the opposite holds for high percentiles, which are generally above their corresponding values of the unconditional distribution. As a consequence, skewness of the conditional distribution increases, thereby shifting some probability mass into the region R_4. LMT represents this behavior well: when conditioning on the 5% quantiles of the statistic η_t, the probability mass falling into R_1 decreases from 40% to 29.3%. The difference of 10.7%, together with a decrease of 3.5% in R_3, goes almost completely in favor of R_4, which increases by 13.8% and reaches the value of 53.8%. Mean and skewness increase taking values of 0.139 resp. 0.241 in the conditional case. In the opposite situation, when conditioning on the 90% quantile of η_t, there is an exchange of probability mass from R_4 together with R_2 into R_1: R_4 and R_2 cede together 7.8% of probability mass, the majority of which moves into the left part of the distribution and increases R_1 by 6%. The mean of the conditional distribution moves away from its unconditional counterpart and becomes negative. Skewness passes from its unconditional value of 0.215 to -1.013. For the majority of the remaining companies the results are similar. Obviously, in some special cases, the differences between the unconditional and conditional distribution are weaker and less indicative. This can be seen on Table 3.6 where we summarize our result: for all statistic quantities analyzed (percentiles, mean, skewness, variance, kurtosis and probability mass falling into the regions R_i), we compute the absolute difference with their corresponding unconditional counterparts and calculate the average changes across the companies analyzed. On average, percentiles of the conditional distributions tend to be above/below the unconditional ones after days belonging to $\mathcal{T}_\alpha^{\text{low}}/\mathcal{T}_\alpha^{\text{high}}$. The average changes for the mean and variance are positive/negative when η_t is low/high. With respect to the probability mass falling into the regions R_i, the upper region R_4 increases when η_t is low. The corresponding probability mass comes form R_1 and R_3 which decrease in value. In the opposite case, when η_t is high, R_4 and R_2 cede some of their probability mass, the majority of which goes in favor of R_1 thereby increasing its value. All the given results support the conjecture that the statistic η_t has predictive power on the future values of the idiosyncratic noise z_t. At the bottom of Table 3.6, we present the results of a simulation study. This simulation aims to analyze whether our results are coincidental or indeed due to the predictive power of our statistic. The main idea is to compute the differences in percentiles between the unconditional distribution of

$\bar{z}_t$ and the "conditional" distribution based on 260 randomly chosen days in our sample [5]. Therefore we simply substitute the values computed for days in $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$ with arbitrary values of our sample and analyze the consequences. If the differences between the unconditional and "conditional" distribution exhibit the same patterns as when analyzing days belonging to $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$, we have to conclude that the impact of the statistic η_t on the idiosyncratic noise z_t was a coincidence. In Table 3.6 we show the result of this simulation study when repeating $N = 1000$ times this simulation for every company and finally averaging across all companies. The differences obtained in percentiles are negligible. These results confirm therefore that after days in $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$, the distribution of the mean of the statistic $\bar{z}_t$ indeed changes significantly. Our results are therefore not coincidental, but due in fact to the predictive power of the statistic η_t.

3.4.4 Predictive power of the statistic η_t on $\bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}$ resp. $\bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}$.

The analysis of the previous sections suggests that our statistic η_t has an impact on the future values of z_t. In particular, when the filtered imbalance η_t between new issued puts and calls is high, the mean of the idiosyncratic noise z_t over a time-window of five trading days tends to be negative and vice versa when η_t is low. The reason for which the time series $\bar{z}_t$ exhibits such behavior remains an open question. We can think of at least two different scenarios: in the first one, the reason for the negative average after a high value of η_t could be a consequence of the fact that the majority of the five subsequent noises tend to be negative. In the second scenario, the negative mean could be attributed to one (or a few) unusually low negative level, whereas the remaining values could be positive as well. In the case that the statistic η_t is low, the opposite situation could occur. In the first scenario, we would conclude that high values of the statistic η_t are able to predict a series of (small) negative noises z_t. In the second case, a high level of η_t tends to be followed by an extremely low value of z_t. When the statistic η_t reaches a low value, our conclusion is reversed. We find empirical evidence for the second scenario. For both measures of future activities $\bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}$ resp. $\bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}$ we estimate both unconditional distributions (for the minimum and maximum noise z_t in a subsequent trading window of five days) and their counterparts when conditioning on statistic η_t (the

[5] When conditioning on the 10% resp. 90% quantile of η_t, the selected days are approximately 260, since the total observations in our sample corresponds to 2600.

conditional distribution of the minimum for days belonging to $\mathcal{T}_\alpha^{\text{high}}$ and the conditional distribution of the maximum for days in $\mathcal{T}_\alpha^{\text{low}}$). As in the previous section, we compute the same statistic quantities and measure the predictive power of η_t by analyzing the changes between unconditional and conditional distributions. We summarize our results in Table 3.7 where we show the concrete computations for the cases of AMR, ATT and AXA, and in Table 3.8 which displays the average behavior across all companies analyzed. The effect of conditioning on η_t is appreciable. After days with a strong imbalance between new issued put and call options, the minimum resp. maximum of the idiosyncratic noise z_t over the following five trading days tends to change probability distribution. After days belonging to $\mathcal{T}_\alpha^{\text{high}}$, an extremely low value for z_t is more likely to occur than in the unconditional case. The opposite holds for days belonging to $\mathcal{T}_\alpha^{\text{low}}$, which tend to be followed by a higher value of z_t than in their unconditional counterpart. Here we briefly discuss some results found for AXA. Corresponding Tables 3.7 provides numerical details and additional interpretations for the remaining companies. When observing the distribution of $\bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}$ after days belonging to $\mathcal{T}_\alpha^{\text{low}}$, the (high) percentiles tend to be above their unconditional counterparts thus shifting the mean to the right and moving a significant probability mass from R_1 to R_4, thereby increasing its value to 52.80%. In the conditional case, values of $\bar{z}_t$ over the 60% percentiles of the unconditional distribution are therefore likely to occur 12.80% more often than during normal times. When looking at $\bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}$, the opposite behavior can be observed after days belonging to $\mathcal{T}_\alpha^{\text{high}}$: when η_t is at its 90% quantile, R_4 is ceding 11.9% of its probability mass, the majority of which goes in favor of R_1. Its value increases by 8.8%, making low values of $\bar{z}_t$ more likely to occur in the conditional case than in the unconditional case. Low percentiles of the conditional distribution tend to be lower than their unconditional counterparts. We report the average changes across the different companies in Table 3.8. After days in $\mathcal{T}_\alpha^{\text{low}}$, the probability of $\bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}$ exceeding the 60% percentile of its unconditional distribution increases on average by 10%, whereas values under the unconditional 40% percentiles are less likely to occur. In the opposite case, after days in $\mathcal{T}_\alpha^{\text{high}}$ $\bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}$ tends to reach values under the 40% unconditional percentile with an average probability of 51%. Such values are therefore 11% more likely to occur in the conditional case than in the unconditional scenario. All these results support the conjecture that days with a strong imbalance between new issued put and call options are likely to be followed by an unusually low idiosyncratic noise in case of $\mathcal{T}_\alpha^{\text{high}}$, respectively high level after days belonging to $\mathcal{T}_\alpha^{\text{low}}$. In order to test the reliability of our results and the predictive power of statistic η_t, we repeat the same simulation study as proposed in the previous section for the case of $\bar{z}_t := \min\{z_{t+1}, ..., z_{t+n}\}$ resp. $\bar{z}_t := \max\{z_{t+1}, ..., z_{t+n}\}$. We report these results at the bottom of Table 3.8 and indicate that after days belonging to $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$, the distributions of the statistics $\bar{z}_t$ indeed change significantly. Our results are therefore not coincidental, but due in fact to the predictive power of the statistic η_t.

3.5 Conclusion

In this paper we studied the informational content of daily changes in open interest. We first defined a daily statistic which represents the imbalance between new issued put and call options. Second, we computed three different measures for future market activities: the mean, the maximum and the minimum of the idiosyncratic noise of the return process over a time-window of 5 trading days. We then analyzed the predictive power of our statistic on these three measures. We found sufficient informational content of our statistic: when conditioning on high/low levels of imbalances between new puts and calls, the distribution functions of all three measures showed significant changes compared to their unconditional counterparts. These differences turned out to be more pronounced when looking at the minimum and maximum idiosyncratic noise over the following 5 trading days: when the number of new issued put options is large compared to the number of new calls, the conditional distribution function becomes heavier on the left side and a large drop in the idiosyncratic return noise is more likely to follow. In the opposite scenario, when the statistic exhibits a large imbalance in favor of call options, the idiosyncratic return noise tends to be higher than after calm days. Our findings confirm the informational content of large daily changes in open interest. Results based on high frequency data are likely to lead to deeper and more detailed results, enhancing our understanding of how new information is possibly reflected in asset prices and how option market variables and subsequent price movements are linked to each other.

		$\hat{a}_0$	$\hat{b}_1$	$\hat{c}_1$	$\hat{d}_1$	$\hat{b}_2$	$\hat{c}_2$	$\hat{d}_2$	F-test			$\hat{a}_0$	$\hat{b}_1$	$\hat{c}_1$	$\hat{d}_1$	$\hat{b}_2$	$\hat{c}_2$	$\hat{d}_2$	F-test
AMR	*value*	0.529	0.017	-0.701	0.007	0.185	-0.285	-0.039	3.321	**KO**	*value*	0.506	-1.461	7.938	0.005	-0.038	0.700	0.001	6.224
	p-value	0.000	0.778	0.038	0.725	0.002	0.397	0.049	0.003		*p-value*	0.000	0.000	0.196	0.799	0.877	0.909	0.981	0.000
ATT	*value*	0.506	-0.272	-1.902	-0.030	0.011	-0.922	-0.011	1.595	**LMT**	*value*	0.503	-0.520	-5.017	-0.037	-0.240	-2.223	0.034	3.348
	p-value	0.000	0.027	0.154	0.136	0.926	0.489	0.582	0.144		*p-value*	0.000	0.005	0.122	0.060	0.191	0.493	0.083	0.003
AXA	*value*	0.482	0.088	0.529	0.021	-0.116	4.037	-0.008	0.734	**MER**	*value*	0.577	-0.698	-9.717	-0.104	-0.294	-4.675	-0.042	12.016
	p-value	0.000	0.588	0.840	0.436	0.473	0.125	0.763	0.623		*p-value*	0.000	0.000	0.001	0.000	0.070	0.114	0.030	0.000
BA	*value*	0.514	-0.344	-1.195	-0.035	0.191	-6.975	-0.008	4.250	**MMC**	*value*	0.494	-0.624	3.942	-0.015	-0.032	-9.203	0.016	5.090
	p-value	0.000	0.006	0.575	0.073	0.131	0.001	0.675	0.000		*p-value*	0.000	0.000	0.074	0.436	0.850	0.000	0.425	0.000
BAC	*value*	0.597	-0.677	-2.368	-0.110	-0.117	3.467	-0.046	10.403	**MO**	*value*	0.488	-0.331	-4.848	0.028	0.281	1.102	0.008	2.542
	p-value	0.000	0.000	0.437	0.000	0.366	0.255	0.018	0.000		*p-value*	0.000	0.027	0.050	0.151	0.061	0.657	0.696	0.019
C	*value*	0.565	-0.900	-6.478	-0.123	-0.448	2.272	0.001	14.948	**MON**	*value*	0.476	-0.125	3.614	-0.043	0.012	-0.409	0.068	9.334
	p-value	0.000	0.000	0.012	0.000	0.003	0.378	0.966	0.000		*p-value*	0.000	0.001	0.000	0.116	0.728	0.542	0.012	0.000
DAL	*value*	0.501	-0.242	-1.412	-0.020	-0.046	-0.897	-0.001	1.820	**MWD**	*value*	0.523	-0.453	-2.996	-0.007	-0.015	0.572	-0.033	2.713
	p-value	0.000	0.029	0.054	0.324	0.676	0.221	0.956	0.092		*p-value*	0.000	0.001	0.183	0.735	0.912	0.799	0.093	0.013
HPQ	*value*	0.462	-0.152	-0.775	0.044	0.297	-4.000	0.031	2.949	**ROK**	*value*	0.483	-0.451	-0.601	-0.001	-0.241	-0.506	0.006	2.021
	p-value	0.000	0.302	0.687	0.024	0.044	0.038	0.111	0.007		*p-value*	0.000	0.002	0.224	0.957	0.105	0.306	0.778	0.060
JPM	*value*	0.517	-0.614	-2.544	-0.064	0.218	0.001	0.015	7.050	**UAL**	*value*	0.458	0.024	-0.544	0.037	-0.180	-0.812	0.046	3.179
	p-value	0.000	0.000	0.027	0.001	0.083	1.000	0.440	0.000		*p-value*	0.000	0.812	0.110	0.113	0.073	0.017	0.051	0.004
KLM	*value*	0.459	-0.495	-1.592	0.053	-0.020	-0.931	0.022	2.580	**UTX**	*value*	0.487	0.077	4.239	0.001	-0.045	3.711	0.002	2.109
	p-value	0.000	0.001	0.405	0.041	0.897	0.626	0.389	0.017		*p-value*	0.000	0.607	0.011	0.957	0.763	0.027	0.920	0.049

Table 3.1. Filtration procedure for the statistic $\mathcal{I}_t$: $\mathcal{I}_t = a_0 + \sum_{j=1}^{2}(b_j r_{t-j} + c_j r_{t-j}^2 + d_j \mathcal{I}_{t-j}) + \eta_t$. The innovations η_t represent the (transformed) imbalance between new issued put and call options at day t which cannot be attributed to stock returns and volatility trends (approximated by the squared returns) of the last n trading days. The estimated constant term $\hat{a}_0$ is around 0.5 and in all cases statistically significant. In the majority of cases, the coefficients related to the one lag past returns, $\hat{b}_1$, are negative and significant at the 5% level. This could be an indication that in periods of bearish/bullish times, investors tend to augment their demand for protective puts/calls with a delay of one day. As a consequence, a day with a high/low return increases/decreases the level of the statistic $\mathcal{I}_t$ in the following day. The coefficient of the two-day lagged return does not seem to have a significant impact on the statistic: it is significant in some cases but with a changing sign. With respect to past squared returns, the statistic $\mathcal{I}_t$ does not react uniformly across the companies analyzed. The same conclusions hold for the lagged statistic values. Overall, the null hypothesis that the R^2-statistic of the regression model (3.3) is equal to zero is rejected due to the high $F-$test.

	$\hat{a}$	*t-stat*	$\hat{b}$	*t-stat*	$\hat{\omega}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\gamma}$
AMR	0.0004	0.5194	1.4431	22.2172	0.0000	0.9675	0.0047	0.0447
ATT	-0.0006	-1.3389	0.9178	23.5497	0.0000	0.8679	0.1001	0.0639
AXA	0.0003	0.6842	1.5150	33.9267	0.0000	0.9442	0.0423	0.0171
BA	0.0002	0.5636	0.8684	27.0752	0.0000	0.9819	0.0062	0.0208
BAC	0.0002	0.8012	1.0653	40.6914	0.0000	0.9568	0.0426	0.0000
C	0.0003	1.1289	1.4175	54.6943	0.0000	0.9487	0.0402	0.0221
DAL	-0.0014	-1.9927	1.2682	21.6157	0.0000	0.9637	0.0000	0.0577
HPQ	0.0001	0.1270	1.3615	33.1175	0.0000	0.9813	0.0112	0.0046
JPM	0.0000	-0.0285	1.4395	47.6717	0.0000	0.9686	0.0299	0.0031
KLM	-0.0008	-1.2120	0.6732	13.0022	0.0000	0.8913	0.0648	0.0435
KO	0.0000	-0.1452	0.6141	23.4202	0.0000	0.9360	0.0409	0.0460
LMT	0.0003	0.8492	0.4178	12.9810	0.0000	0.9831	0.0111	0.0115
MER	0.0005	1.3333	1.5757	50.9392	0.0000	0.9699	0.0213	0.0174
MMC	0.0001	0.4466	1.0225	36.2551	0.0000	0.8004	0.0993	0.0000
MON	0.0012	2.0802	0.6732	13.7436	0.0000	0.9340	0.0424	0.0222
MWD	0.0004	1.0488	1.7076	52.0687	0.0000	0.9336	0.0290	0.0624
ROK	0.0002	0.3990	0.8786	22.2236	0.0000	0.9520	0.0000	0.0960
UAL	-0.0013	-1.2928	1.0383	12.7934	0.0001	0.7129	0.1112	0.2960
UTX	0.0005	1.6596	0.9553	35.0925	0.0000	0.9840	0.0160	0.0000
MO	0.0004	0.9509	0.5022	14.5554	0.0000	0.9497	0.0189	0.0536

Table 3.2. Extrapolation of the return innovations z_t using the asymmetric GARCH specification as described in equation (3.4): $r_t = a + b\, r_{M,t} + \varepsilon_t$, $\varepsilon_t = \sigma_t z_t$, $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma J_{t-1}\varepsilon_{t-1}^2$. The coefficient $\hat{a}$ does not statistically differ from zero, whereas $\hat{\beta}$ is significant for all companies and takes values between 0.4 and 1.7. The coefficient $\hat{\gamma}$ related to the leverage effect in (3.4) is positive for almost all companies.

	$\hat{\alpha}_0$	$\hat{\beta}_1$	$\hat{\gamma}_1$	$\hat{\beta}_2$	$\hat{\gamma}_2$	$\hat{\beta}_3$	$\hat{\gamma}_3$	$\hat{\beta}_4$	$\hat{\gamma}_4$	$\hat{\beta}_5$	$\hat{\gamma}_5$	F-test
AMR	-0.427	-0.663	-0.149	0.562	1.203	1.383	0.365	-0.770	-1.742	-0.338	2.416	1.313
ATT	-0.281	-0.221	0.423	0.300	-0.338	-0.299	0.399	-0.215	0.120	1.310	0.646	0.288
AXA	0.511	1.525	-0.989	-0.512	-0.010	-0.251	-0.958	0.242	-0.632	-0.400	1.309	0.673
BA	-0.405	0.409	-0.159	-0.106	0.704	0.185	0.404	-0.075	0.150	1.940	0.238	0.463
BAC	-0.067	0.825	0.241	0.290	-0.149	-0.003	0.364	-0.103	-0.702	1.074	0.486	0.248
C	0.486	0.900	-0.351	0.502	0.473	0.602	-1.273	-0.096	0.212	2.150	-1.045	0.765
DAL	-0.789	-0.003	0.514	0.972	0.718	0.485	1.122	-0.118	1.256	-0.744	1.778	0.990
HPQ	-0.163	0.062	0.635	-0.370	-0.942	-0.141	0.907	0.915	-0.031	-0.850	0.102	0.389
JPM	-0.275	1.884	0.848	-0.525	0.256	0.601	0.798	1.549	0.967	0.356	-1.229	0.945
KLM	-0.335	-0.239	1.686	-0.313	-0.741	-0.143	0.181	0.103	-1.102	0.946	1.147	0.691
KO	-0.891	0.659	0.176	-0.009	1.341	0.147	-0.330	0.435	0.300	2.878	1.084	1.212
LMT	0.216	0.195	-1.400	-0.194	0.218	-0.214	0.644	0.409	-1.154	-1.026	0.847	0.509
MER	0.090	2.110	-1.183	-0.632	0.783	-0.118	-0.294	-0.635	1.455	1.165	-1.023	1.031
MMC	-0.135	1.178	0.174	0.355	-0.489	0.480	-0.447	-0.126	0.893	-0.650	0.905	0.554
MON	-0.448	0.560	1.778	-1.876	-1.385	1.348	1.088	0.764	0.284	0.794	-0.330	1.092
MWD	-0.541	0.732	1.588	-0.306	1.156	-0.097	1.496	-0.004	-1.796	0.796	-0.568	1.141
ROK	0.026	-0.531	-0.130	-0.632	-0.553	0.054	0.066	0.909	0.509	-0.467	-0.225	0.182
UAL	-0.538	-0.024	0.666	0.157	-1.357	2.013	3.709	0.875	0.330	0.321	0.068	1.784
UTX	-0.180	0.873	0.506	0.255	0.154	0.598	0.536	0.910	0.267	0.382	-0.362	0.242
MO	0.752	0.427	-1.496	0.452	-0.285	-0.264	0.313	-0.159	0.227	-1.473	-1.328	0.690

Table 3.3. Causality direction test: we first use the regression model $\eta_t = a_0 + \sum_{j=1}^{5} b_j \eta_{t-j} + \epsilon_t$ in order to remove possible autocorrelations in η_t which could bias the results. With the model $\epsilon_t = \alpha_0 + \sum_{j=1}^{5} (\beta_j z_{t-j} + \gamma_j z_{t-j}^2) + \mu_t$ we investigate whether past returns innovations influence the future development of the statistic η_t. The corresponding t-statistics and F-test for the null hypothesis of insignificant regression coefficients are reported. Overall, past stock returns innovations do not appear to explain future values of the filtered open interest statistic.

	Unconditional mean		*Conditional mean*							
			Q-5%		**Q-10%**		**Q-90%**		**Q-95%**	
	mean	*p-value*	*mean*	*p-value*	*mean*	*p-value*	*mean*	*p-value*	*mean*	*p-value*
AMR	-0.019	0.029	0.042	0.041	0.071	0.055	-0.125	0.039	-0.107	0.048
ATT	0.001	0.919	0.054	0.001	0.076	0.035	-0.063	0.096	-0.081	0.044
AXA	-0.002	0.794	0.071	0.016	0.059	0.009	-0.035	0.155	-0.024	0.454
BA	0.001	0.879	0.122	0.000	0.101	0.000	0.082	0.002	0.084	0.018
BAC	-0.002	0.775	-0.016	0.523	0.034	0.062	-0.004	0.847	-0.003	0.912
C	-0.003	0.588	0.017	0.448	0.022	0.362	-0.025	0.144	-0.034	0.185
DAL	-0.002	0.837	0.063	0.057	0.072	0.003	-0.034	0.138	-0.044	0.220
HPQ	-0.002	0.812	-0.010	0.152	-0.075	0.268	-0.119	0.000	-0.101	0.002
JPM	0.001	0.882	-0.001	0.981	0.040	0.077	0.022	0.347	0.034	0.253
KLM	-0.005	0.648	-0.033	0.021	-0.021	0.042	-0.075	0.034	-0.122	0.017
KO	0.000	0.968	0.052	0.054	0.055	0.005	-0.039	0.047	-0.005	0.860
LMT	0.002	0.701	0.139	0.000	0.112	0.000	-0.115	0.000	-0.096	0.005
MER	-0.002	0.796	-0.027	0.327	-0.005	0.811	0.029	0.157	0.004	0.891
MMC	-0.003	0.668	0.077	0.008	0.078	0.000	-0.022	0.079	-0.008	0.788
MON	0.003	0.769	-0.078	0.031	-0.054	0.023	-0.037	0.062	-0.057	0.016
MWD	-0.004	0.584	-0.013	0.679	0.018	0.393	0.009	0.635	0.025	0.380
ROK	-0.003	0.748	0.026	0.011	0.060	0.021	-0.020	0.039	-0.047	0.083
SPX	0.000	0.463	-0.015	0.485	0.000	0.669	0.000	0.197	0.000	0.137
UAL	0.008	0.430	0.081	0.053	-0.078	0.008	0.032	0.072	0.058	0.042
UTX	-0.004	0.524	-0.053	0.028	0.019	0.318	-0.019	0.293	-0.003	0.906
MO	0.000	0.945	-0.014	0.048	-0.010	0.047	0.073	0.004	0.086	0.028

Table 3.4. Regression analysis: z_t^{mean} when conditioning on several levels of η_t. We test the null hypothesis that filtered η_t has no impact on the filtered stock returns z_t: $H_0 : E[\bar{z}_t|\eta_t > q_\alpha^\eta] = 0$, where q_α^η is α-quantile of η_t. We can test the null hypothesis by simply regressing the conditional innovations, $\bar{z}_t|\eta_t > q_\alpha^\eta$, on a constant, and testing whether the constant is significantly away from zero. We also study whether low values of the filtered statistic η_t have an impact on future idiosyncratic shocks, testing $H_0 : E[\bar{z}_t|\eta_t < q_\alpha^\eta] = 0$. We report the conditional average of the innovations and the p-values of the regressions for each stock in our database when conditioning on η_t takes place. With respect to the unconditional mean, all values are statistically significant non-different from zero (high p-values). The conditional means tend to exhibit a different behavior: after days belonging to $\mathcal{T}_\alpha^{\mathrm{high}}$, the mean of the innovations computed over a window of five trading days tends to be negative and vice versa when we look at days in $\mathcal{T}_\alpha^{\mathrm{low}}$.

	percentiles									descriptive statistics				cond. vs. uncond. distribution			
	10%	20%	30%	40%	50%	60%	70%	80%	90%	mean	variance	skewness	kurtosis	R_1	R_2	R_3	R_4
AMR	-0.508	-0.324	-0.210	-0.117	-0.021	0.074	0.180	0.306	0.502	-0.019	0.195	-0.441	7.105	40.00%	10.00%	10.00%	40.00%
Q-5%	-0.338	-0.214	-0.114	-0.063	-0.003	0.089	0.206	0.299	0.522	0.042	0.146	-0.184	5.140	30.20%	15.50%	13.30%	41.00%
Q-10%	-0.316	-0.205	-0.112	-0.063	-0.004	0.090	0.206	0.332	0.592	0.071	0.163	0.798	6.740	29.70%	16.70%	12.00%	41.60%
Q-90%	-0.667	-0.474	-0.347	-0.225	-0.082	0.005	0.091	0.255	0.371	-0.125	0.255	-2.047	10.981	48.30%	8.00%	12.10%	31.60%
Q-95%	-0.606	-0.451	-0.312	-0.179	-0.036	0.038	0.098	0.261	0.369	-0.107	0.256	-2.114	11.016	45.00%	6.90%	13.90%	34.20%
ATT	-0.413	-0.251	-0.149	-0.068	0.005	0.066	0.148	0.250	0.421	0.001	0.149	-0.083	6.520	40.00%	10.00%	10.00%	40.00%
Q-5%	-0.382	-0.205	-0.117	-0.053	0.027	0.135	0.195	0.314	0.542	0.054	0.170	-0.388	6.799	36.70%	12.30%	5.00%	46.00%
Q-10%	-0.333	-0.176	-0.105	-0.053	-0.002	0.106	0.182	0.315	0.552	0.076	0.173	0.647	7.130	36.10%	15.80%	5.20%	42.90%
Q-90%	-0.595	-0.334	-0.209	-0.126	0.001	0.058	0.142	0.243	0.383	-0.063	0.166	-0.322	4.753	43.80%	7.30%	10.10%	38.80%
Q-95%	-0.702	-0.424	-0.244	-0.155	0.001	0.068	0.157	0.263	0.408	-0.081	0.193	-0.619	3.945	45.50%	6.70%	6.40%	41.40%
DAL	-0.498	-0.318	-0.182	-0.087	0.003	0.093	0.198	0.322	0.497	-0.002	0.189	-0.911	10.970	40.00%	10.00%	10.00%	40.00%
Q-5%	-0.404	-0.232	-0.124	-0.041	0.069	0.168	0.270	0.355	0.512	0.063	0.132	0.130	3.029	36.00%	8.10%	8.30%	47.60%
Q-10%	-0.377	-0.225	-0.136	-0.045	0.037	0.129	0.265	0.360	0.520	0.072	0.139	0.529	3.847	35.30%	9.70%	9.90%	45.10%
Q-90%	-0.459	-0.308	-0.200	-0.112	-0.041	0.036	0.143	0.204	0.406	-0.034	0.129	0.047	3.560	44.70%	12.60%	7.90%	34.80%
Q-95%	-0.571	-0.332	-0.212	-0.138	-0.039	0.065	0.157	0.270	0.491	-0.044	0.159	-0.220	3.018	45.10%	11.50%	5.60%	37.80%
LMT	-0.386	-0.233	-0.146	-0.067	-0.001	0.071	0.151	0.240	0.385	0.002	0.107	0.215	5.494	40.00%	10.00%	10.00%	40.00%
Q-5%	-0.237	-0.133	-0.064	0.004	0.137	0.217	0.262	0.436	0.551	0.139	0.107	0.241	3.040	29.30%	10.40%	6.50%	53.80%
Q-10%	-0.228	-0.133	-0.090	-0.016	0.049	0.177	0.248	0.395	0.532	0.112	0.097	0.423	3.155	31.90%	12.10%	8.30%	47.70%
Q-90%	-0.599	-0.381	-0.228	-0.130	-0.031	0.040	0.097	0.147	0.274	-0.115	0.138	-1.013	4.940	46.00%	6.60%	11.80%	35.60%
Q-95%	-0.706	-0.382	-0.202	-0.106	-0.020	0.060	0.117	0.190	0.303	-0.096	0.148	-0.881	4.017	44.60%	6.20%	10.70%	38.50%
MWD	-0.404	-0.274	-0.181	-0.086	-0.008	0.071	0.157	0.254	0.396	-0.004	0.113	0.302	4.651	40.00%	10.00%	10.00%	40.00%
Q-5%	-0.391	-0.266	-0.173	-0.089	-0.040	0.054	0.091	0.226	0.432	-0.013	0.125	0.466	5.428	40.10%	16.00%	10.80%	33.10%
Q-10%	-0.326	-0.221	-0.147	-0.064	-0.013	0.068	0.151	0.245	0.409	0.018	0.111	0.261	5.469	36.10%	15.40%	9.10%	39.40%
Q-90%	-0.407	-0.267	-0.170	-0.092	-0.006	0.073	0.155	0.239	0.379	-0.009	0.100	-0.049	3.798	40.80%	8.90%	9.90%	40.40%
Q-95%	-0.389	-0.259	-0.175	-0.105	-0.028	0.042	0.132	0.236	0.336	-0.025	0.101	-0.250	4.179	41.60%	10.00%	10.80%	37.60%

Table 3.5. Distribution of z_t^{mean} when conditioning on several levels of η_t. In the first column we report the level of conditioning along with the company's name. The unconditional distribution $F^{\text{uncond}}(\bar{z}_t)$ is defined as the benchmark distribution. At the same level as the company's name, information of the unconditional distribution are reported. We divide its domain into four regions: $R_1 := (-\infty, q_{0.40}^{F^{\text{uncond}}}]$, $R_2 := (q_{0.40}^{F^{\text{uncond}}}, q_{0.50}^{F^{\text{uncond}}}]$, $R_3 := (q_{0.50}^{F^{\text{uncond}}}, q_{0.60}^{F^{\text{uncond}}}]$, $R_4 := (q_{0.60}^{F^{\text{uncond}}}, \infty)$, where $q_\alpha^{F^{\text{uncond}}}$ denotes the α-quantile of the unconditional distributions. From the second to the ninth column, we report the percentiles of the estimated distribution. The next four columns report some summary statistics of these distributions, and the last four give an overview of the probability mass falling into the regions R_i, $i = 1, ..., 4$.

Average results for conditional mean

	percentiles								
	10%	20%	30%	40%	50%	60%	70%	80%	90%
Q-5%	0.087	0.072	0.058	0.034	0.023	0.028	0.041	0.062	0.089
Q-10%	0.114	0.081	0.057	0.034	0.020	0.031	0.047	0.075	0.093
Q-90%	-0.124	-0.067	-0.045	-0.029	-0.009	-0.030	-0.057	-0.077	-0.116
Q-95%	-0.116	-0.094	-0.053	-0.037	-0.008	-0.026	-0.042	-0.065	-0.085

	descriptive statistics				cond. vs. uncond. distribution			
	mean	variance	skewness	kurtosis	R_1	R_2	R_3	R_4
Q-5%	0.062	-0.005	0.771	-5.426	-6.32%	4.87%	-1.19%	2.64%
Q-10%	0.076	-0.012	1.009	-5.130	-6.55%	4.10%	-0.96%	3.41%
Q-90%	-0.078	-0.001	-0.599	-3.856	5.55%	-2.39%	2.28%	-5.44%
Q-95%	-0.082	0.019	-0.400	-5.663	4.27%	-2.90%	2.39%	-3.76%

Robustness checks for mean

percentiles								
10%	20%	30%	40%	50%	60%	70%	80%	90%
4.4E-06	0.000765	0.000422	-0.00017	0.000315	-0.00036	0.000049	-0.00063	0.000392

Table 3.6. Summary of distribution of z_t^{mean} when conditioning on several levels of η_t. We report the average differences across companies between the statistical quantities (percentiles, probability mass falling into the regions R_1, ..., R_4 and key values as the mean, variance, skewness and kurtosis) when moving from the unconditional to the conditional distribution. The robustness check consists of computing the differences in percentiles between the unconditional distribution of $\bar{z}_t$ and the "conditional" distribution based on 260 randomly chosen days in our sample. Therefore, we simply substitute the values computed for days in $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$ with arbitrary values of our sample and analyze the consequences.

	percentiles									descriptive statistics				cond. vs. uncond. distribution			
	10%	20%	30%	40%	50%	60%	70%	80%	90%	mean	variance	skewness	kurtosis	R_1	R_2	R_3	R_4
AMR, max	0.25	0.47	0.63	0.78	0.96	1.13	1.37	1.62	2.00	1.08	0.63	1.52	8.13	40.00%	10.00%	10.00%	40.00%
Q-5%	0.47	0.58	0.78	0.98	1.14	1.31	1.51	1.60	1.79	1.15	0.32	0.35	2.76	28.80%	10.90%	9.20%	51.10%
Q-10%	0.47	0.60	0.72	0.98	1.11	1.31	1.47	1.62	1.87	1.16	0.41	1.10	5.71	32.50%	7.30%	10.50%	49.70%
AMR, min	-1.86	-1.51	-1.26	-1.13	-0.98	-0.83	-0.67	-0.53	-0.34	-1.09	0.81	-6.84	93.54	40.00%	10.00%	10.00%	40.00%
Q-90%	-2.10	-1.69	-1.48	-1.27	-1.17	-0.98	-0.82	-0.66	-0.44	-1.33	1.85	-6.94	65.35	51.10%	8.20%	10.00%	30.70%
Q-95%	-2.13	-1.68	-1.52	-1.30	-1.10	-0.95	-0.80	-0.65	-0.43	-1.30	1.83	-7.10	68.21	49.10%	7.80%	11.50%	31.60%
ATT, max	0.23	0.40	0.51	0.63	0.77	0.92	1.15	1.41	1.83	0.95	0.62	2.42	13.75	40.00%	10.00%	10.00%	40.00%
Q-5%	0.40	0.50	0.65	0.81	0.87	1.14	1.40	1.61	2.02	1.11	0.64	1.82	8.26	29.20%	10.00%	14.20%	46.60%
Q-10%	0.33	0.47	0.60	0.72	0.86	1.05	1.31	1.59	2.06	1.12	0.97	2.86	14.62	32.70%	11.50%	12.00%	43.80%
ATT, min	-1.64	-1.26	-1.05	-0.91	-0.75	-0.61	-0.50	-0.38	-0.23	-0.93	0.81	-4.45	34.37	40.00%	10.00%	10.00%	40.00%
Q-90%	-2.10	-1.47	-1.17	-0.98	-0.88	-0.75	-0.61	-0.47	-0.28	-1.09	0.94	-3.16	18.01	47.40%	13.00%	9.30%	30.30%
Q-95%	-2.22	-1.52	-1.19	-1.02	-0.93	-0.79	-0.67	-0.55	-0.27	-1.20	1.29	-3.13	16.22	52.10%	12.50%	9.70%	25.70%
AXA, max	0.25	0.45	0.58	0.68	0.81	0.91	1.08	1.30	1.60	0.88	0.29	0.86	4.26	40.00%	10.00%	10.00%	40.00%
Q-5%	0.47	0.65	0.76	0.83	0.95	1.15	1.30	1.49	1.75	1.05	0.22	0.41	2.61	26.40%	11.90%	8.90%	52.80%
Q-10%	0.42	0.63	0.74	0.82	0.92	1.08	1.25	1.49	1.75	1.03	0.26	0.81	4.37	27.20%	11.10%	9.80%	51.90%
AXA, min	-1.72	-1.30	-1.01	-0.91	-0.79	-0.66	-0.56	-0.40	-0.28	-0.88	0.31	-0.95	3.80	40.00%	10.00%	10.00%	40.00%
Q-90%	-1.79	-1.32	-1.19	-0.96	-0.89	-0.74	-0.67	-0.54	-0.39	-0.98	0.27	-1.07	4.12	48.80%	9.00%	14.10%	28.10%
Q-95%	-1.74	-1.30	-1.15	-0.94	-0.83	-0.68	-0.65	-0.54	-0.39	-0.96	0.30	-1.29	4.55	45.50%	8.90%	13.30%	32.30%

Table 3.7. Distribution of $z_t^{\min}$, resp. $z_t^{\max}$ when conditioning on several levels of η_t. In the first column we report the level of conditioning along with the company's name. The unconditional distribution $F^{\text{uncond}}(\bar{z}_t)$ is defined as the benchmark distribution. At the same level as the company's name, information on the unconditional distribution is reported. We divide its domain into four regions: $R_1 := (-\infty, q_{0.40}^{F^{\text{uncond}}}]$, $R_2 := (q_{0.40}^{F^{\text{uncond}}}, q_{0.50}^{F^{\text{uncond}}}]$, $R_3 := (q_{0.50}^{F^{\text{uncond}}}, q_{0.60}^{F^{\text{uncond}}}]$, $R_4 := (q_{0.60}^{F^{\text{uncond}}}, \infty)$, where $q_{\alpha}^{F^{\text{uncond}}}$ denotes the α-quantile of the unconditional distributions. From the second to the ninth column, we report the percentiles of the estimated distribution. The next four columns report some summary statistics of these distributions, and the last four give an overview of the probability mass falling into the regions R_i, $i = 1, ..., 4$.

Average results for conditional min/max

	percentiles								
	10%	20%	30%	40%	50%	60%	70%	80%	90%
Q-5%	0.123	0.130	0.142	0.152	0.158	0.171	0.175	0.210	0.186
Q-10%	0.127	0.122	0.133	0.135	0.140	0.161	0.174	0.205	0.192
Q-90%	-0.257	-0.176	-0.178	-0.153	-0.205	-0.131	-0.125	-0.115	-0.107
Q-95%	-0.344	-0.220	-0.200	-0.171	-0.214	-0.142	-0.134	-0.130	-0.116

	descriptive statistics				cond. vs. uncond. distribution			
	mean	variance	skewness	kurtosis	R_1	R_2	R_3	R_4
Q-5%	0.167	0.050	-0.532	-5.556	-10.43%	-0.97%	0.55%	10.85%
Q-10%	0.167	0.094	-0.207	-2.968	-9.89%	-0.79%	0.28%	10.40%
Q-90%	-0.170	0.153	1.867	-37.715	11.05%	0.04%	-0.48%	-10.61%
Q-95%	-0.207	0.356	2.259	-42.478	11.84%	-0.40%	-0.15%	-11.29%

Robustness checks for min/max

	percentiles								
	10%	20%	30%	40%	50%	60%	70%	80%	90%
max	0.0005	-0.0005	0.0015	0.0002	0.0012	0.0024	-0.0004	0.0013	-0.0013
min	0.0003	0.0022	-0.0018	-0.0015	0.0002	0.0001	-0.0003	-0.0017	0.0012

Table 3.8. Summary of distribution of $z_t^{\min}$, resp. $z_t^{\max}$ when conditioning on several levels of η_t. We report the average differences across companies between the statistical quantities (percentiles, probability mass falling into the regions R_1, ..., R_4 and key values as the mean, variance, skewness and kurtosis) when moving from the unconditional to the conditional distribution. The robustness check consists of computing the differences in percentiles between the unconditional distribution of $z_t^{\min}$, resp. $z_t^{\max}$ and the "conditional" distribution based on 260 randomly chosen days in our sample. Therefore, we simply substitute the values computed for days in $\mathcal{T}_\alpha^{\text{low}}$ resp. $\mathcal{T}_\alpha^{\text{high}}$ with arbitrary values of our sample and analyze the consequences.

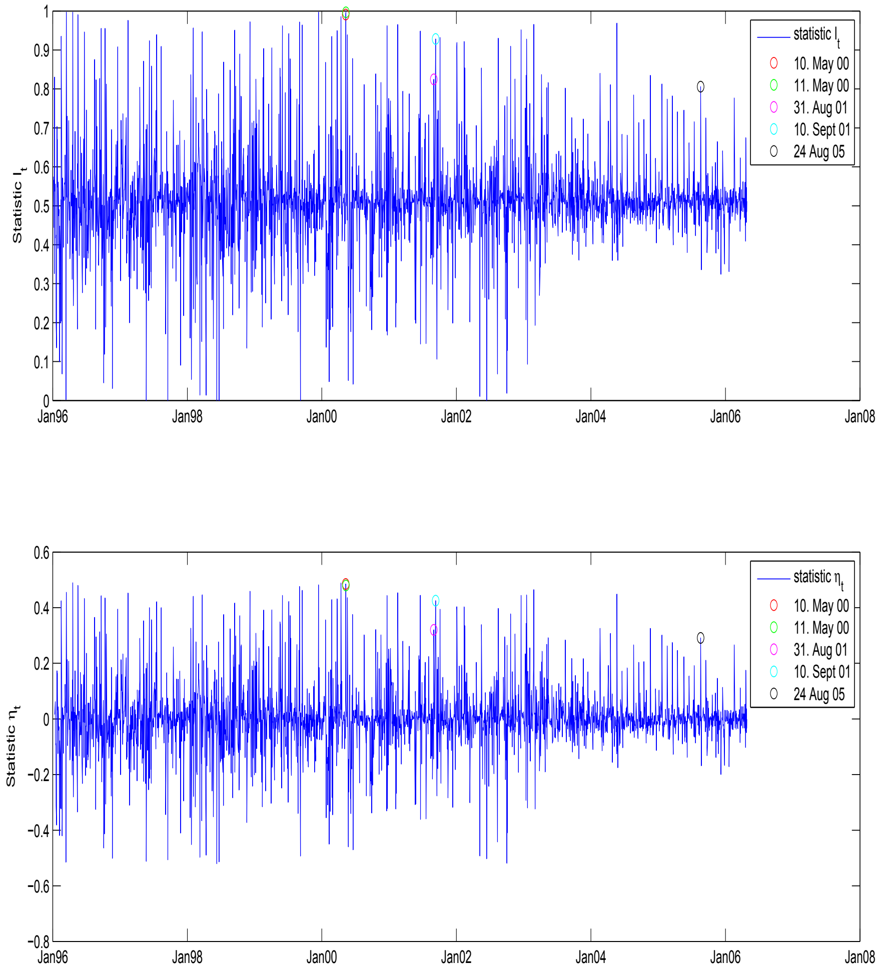

Figure 3.1. Imbalance statistic $\mathcal{I}_t := \Phi\left(\frac{I_t - mean(I_t)}{std(I_t)}\right)$, with $I_t := \frac{OI_{P,t} - OI_{P,t-1}}{OI_{P,t-1}} - \frac{OI_{C,t} - OI_{C,t-1}}{OI_{C,t-1}}$, and statistic innovations η_t, where $\mathcal{I}_t = a_0 + \sum_{j=1}^{n}(b_j\, r_{t-j} + c_j\, r_{t-j}^2 + d_j\, \mathcal{I}_{t-j}) + \eta_t$ for American Airlines (AMR). $OI_{P,t}$ and $OI_{C,t}$ represent the total option open interest at day t across all available maturities and strikes. During the marked days, according to Chesney, Crameri and Mancini (2009), informed trading activities take place on the options market.

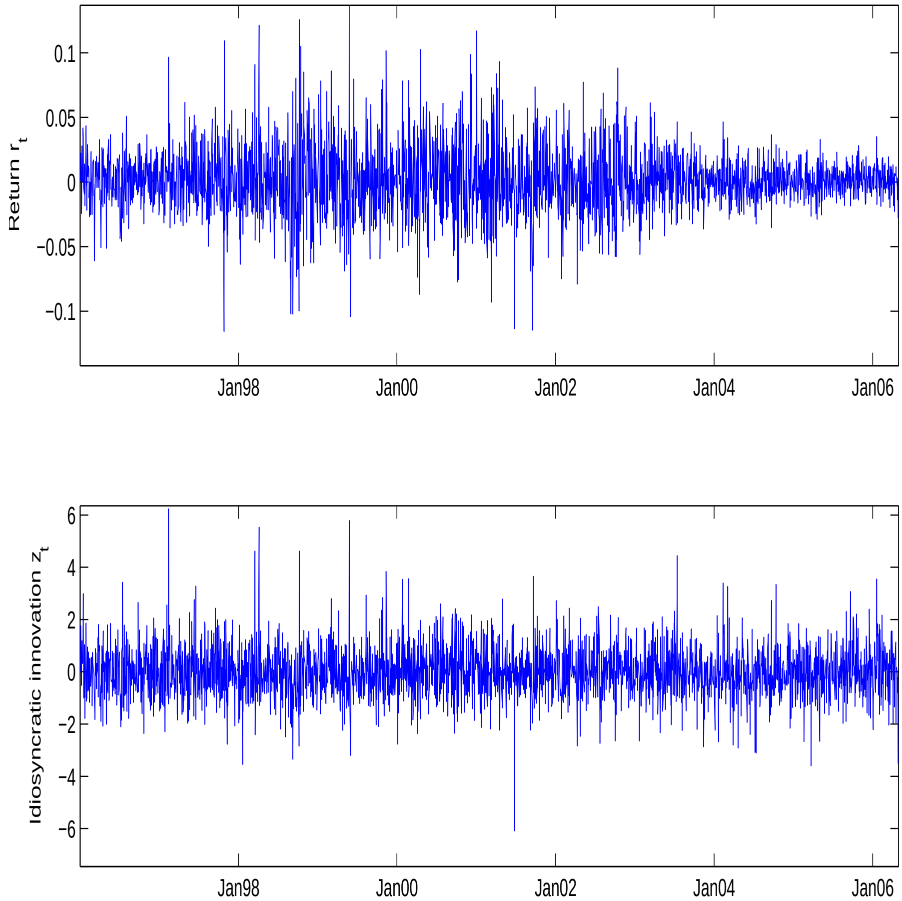

Figure 3.2. The first plot shows the daily raw returns r_t of MER stock. The second plot shows the idiosyncratic noise z_t of MER stock estimated using model (3.4), which filters out market influence and volatility clustering from the raw returns: $r_t = a + b\, r_{M,t} + \varepsilon_t$, $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \gamma J_{t-1}\varepsilon_{t-1}^2$, where $r_{M,t}$ is the market return (thereafter approximated by the S&P 500), $\varepsilon_t = \sigma_t\, z_t$, $z_t \sim f(0,1)$ and $J_{t-1} = 1$, when $\varepsilon_{t-1} < 0$ and $J_{t-1} = 0$, otherwise.

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A Non-parametric estimation of conditional distribution function

Non-parametric regression is often used as an alternative to ordinary least squares regression when the relationship between Y and X exhibits nonlinearity. Although this technique is mostly used for the estimation of the conditional mean of Y given that the exogenous variable X takes on a particular value x, making use of the fact that the expected value of an indicator function equals its probability, Nadaraya and Watson derived an estimator for the conditional distribution function. For our purposes we use the so-called *adjusted Nadaraya-Watson* estimator, which differs slightly from the original estimator by introducing the weights w_i. The main reason for this choice is that the estimator for the conditional distribution $F(y|x)$ must be monotonic in y and takes value in the unit interval, properties which are no longer guaranteed when using the unmodified and original version of the estimator. We introduce the bivariate estimator in the following section.

A.1 The *adjusted Nadaraya-Watson* estimator

Let $\{\mathbf{X}_t, Y_t\}_{t=1}^T$ be observations from a strictly stationarity process where Y_t is a scalar variable and $\mathbf{X}_t := (X_{1,t}, X_{2,t})$ a vector of exogenous variables. Defining $Z_t := \mathbf{1}_{\{Y_t \le y\}}$, the conditional distribution function $F(y|\mathbf{x}) := \mathbb{P}(Y_t \le y|\mathbf{x})$ can be computed as $E[Z_t|\mathbf{X}_t = \mathbf{x}_t]$. Following [16], the *adjusted Nadaraya-Watson* reads

$$\tilde{F}(y|\mathbf{x})^{NW} = \frac{\sum_{t=1}^{T} Z_t w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})}{\sum_{t=1}^{T} w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})}. \tag{3.11}$$

where $\{w_t(\mathbf{x})\}_{t=1}^T$ are chosen to maximize $\prod_{t=1}^{T} w_t(\mathbf{x})$ under the restrictions that $w_t(\mathbf{x}) \ge 0$, $\sum_{t=1}^{T} w_t(\mathbf{x}) = 1$ and $\sum_{t=1}^{T} w_t(\mathbf{x})(X_{m,t} - x_m) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x}) = 0$ for $m = 1, 2$. Here the function $K_{\mathbf{H}}(.)$ denotes a multivariate kernel with bandwidth matrix $\mathbf{H}$, which needs to be specified later on. As noted by [16], it is useful to view the estimator as the local linear estimator of $F(y|\mathbf{x})$ with weights $K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})$ replaced by $w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})$. Therefore we can easily show that the *adjusted Nadaraya-Watson* corresponds to the coefficient a of the following maximization problem derived when using the standard local linear estimator procedure:

$$\max_{a,b} \sum_{t=1}^{T} (Z_t - a - (\mathbf{X}_t - \mathbf{x})\mathbf{b})^2 w_t(\mathbf{x}) K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x}). \tag{3.12}$$

Some properties of this estimator (such as bias and variance) can be found in [20] pp. 96-104.

The implementation of the estimator $\tilde{F}(y|\mathbf{x})$ requires a number of practical issues that must be analyzed. In particular, we need to compute the weights $w_t(\mathbf{x})$ and specify the (bivariate) kernel function $K_{\mathbf{H}}(\mathbf{x}) := |\mathbf{H}|^{-1}K(\mathbf{H}^{-1}\mathbf{x})$. We carry out the computation of $w_t(\mathbf{x})$ by using the standard Lagrange Multiplier methodology. More precisely, given the objective function $\prod_{t=1}^{T} w_t(\mathbf{x})$ and its constraints (see above) the Lagrangian reads

$$\mathcal{L} = \sum_{t=1}^{T} \ln w_t(\mathbf{x}) - \lambda_0\Big(\sum_{t=1}^{T} w_t(\mathbf{x}) - 1\Big) \tag{3.13}$$

$$-\sum_{m=1}^{2} \lambda_m \sum_{t=1}^{T} w_t(\mathbf{x})(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x}). \tag{3.14}$$

Taking partial derivatives with respect to $w_t(\mathbf{x})$, we get the first order condition

$$\frac{1}{w_t(\mathbf{x})} - \lambda_0 - \sum_{m=1}^{2} \lambda_m(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x}) = 0, \tag{3.15}$$

holding for $t = 1, ..., T$.

It can be easily shown (using the conditions $\sum_{t=1}^{T} w_t(\mathbf{x}) = 1$ and $\sum_{t=1}^{T} w_t(\mathbf{x})(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x}) = 0$) that $\lambda_0 = T$. Furthermore, λ_m for $m = 1, 2$ satisfies (eliminating $w_t(\mathbf{x})$ by using the same conditions)

$$\sum_{t=1}^{T} \frac{(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})}{T + \sum_{m=1}^{2} \lambda_m \sum_{t=1}^{T}(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})} = 0. \tag{3.16}$$

These equations are solved numerically. Note that the solution (λ_1, λ_2) depends on the level of conditioning $\mathbf{x}$, even though this was not explicitly expressed in the above calculations. Having now λ_m for $m = 1, 2$, the weights $w_t(\mathbf{x})$ are computed using the first order condition. It follows

$$w_t(\mathbf{x}) = \frac{1}{T + \sum_{m=1}^{2} \lambda_m(X_{m,t} - x_m)K_{\mathbf{H}}(\mathbf{X}_t - \mathbf{x})}, \tag{3.17}$$

for $t = 1, ..., T$.

For the kernel function we choose the bivariate normal distribution $K(\mathbf{x}) := \mathcal{N}_2(0, \mathbf{1})$. It is often noted that the choice of the kernel in smoothing problems is not usually crucial (for details please consult [24]). In contrast, the choice of the bandwidth matrix $\mathbf{H}$ =

$diag(h_1, h_2)$ has an important impact on the results of non-parametric regression. Several procedures have been proposed in the literature. One can choose the bandwidth according to the bivariate Silverman's rule (also known as the bivariate normal reference rule). Specifically, for the exogenous variable i the bandwidth is chosen as $h_i = T^{-1/6}\hat{\sigma}_{X_i}$. Note that this is an objective bandwidth selection procedure which reflects the volatility of each explanatory variable, denoted by $\hat{\sigma}_{X_i}$ for conditioning variable i. Another approach, based on [16], consists of choosing $\mathbf{H}(\mathbf{x}) = h(\mathbf{x})\mathbf{1}_M$, where $\mathbf{1}_n$ is the identity matrix of dimension n. As this procedure uses effectively only one bandwidth for multiple regressors, it is advisable to always scale the regressors to a common variance before the estimator is implemented. The optimal value of $h(\mathbf{x})$ is computed using the bootstrap bandwidth selection method suggested in [16]. Note that this approach uses a variable bandwidth dependent on the level of the conditioning variable $\mathbf{x}$. Based on the conclusions made in [16], we implement the latter approach in this paper and briefly discuss it as follows: Let us define $\mathbf{X}_t := (X_t, Z_t)$. Assume that we want to estimate the conditional distribution of Y on a grid space $(y_1, ..., y_N)$ conditioning on X and Z being for example at one of their $20\%, 40\%, 60\%$ or 80% percentiles. Let us fix the level of conditioning and denote it by $\mathbf{x}_k := (x_k, z_k)$. In the following we explain how the optimal bandwidth $h(\mathbf{x}_k)$ is selected for this specific choice of $\mathbf{x}_k$.

First, we fit a simple parametric model to the observable data $(Y_t, \mathbf{X}_t), t = 1, ..., T$. In this preliminary version, we choose the simple model

$$Y_t = a_0 + a_1 X_t + a_2 Z_t + a_3 X_t Z_t + a_4 X_t^2 + a_5 Z_t^2 + \sigma\epsilon_t, \tag{3.18}$$

where ϵ_t is standard normal and $a_1, ..., a_5, \sigma$ are estimated from the observable data $(Y_t, \mathbf{X}_t)$. Having estimates $\hat{a}_1, ..., \hat{a}_5, \hat{\sigma}$, we compute $\hat{F}(y_n|\mathbf{x}_k)^{par}$ for every grid point $y_n \in (y_1, ..., y_N)$ based on the assumption that $\epsilon_t \sim \mathcal{N}(0, 1)$. This leads to the parametric estimator

$$\hat{F}(y_n|\mathbf{x}_k)^{par} = \Phi(y_n, \hat{\mu}_k, \hat{\sigma}), \tag{3.19}$$

with $\hat{\mu}_k = \hat{a}_0 + \hat{a}_1 x_k + \hat{a}_2 z_k + \hat{a}_3 x_k z_k + \hat{a}_4 x_k^2 + \hat{a}_5 z_k^2$ and $\Phi(y, \mu, \sigma)$ being the cdf of a normal distribution with mean μ and standard deviation σ evaluated at the point y.

By Monte Carlo simulations from this model and using the observations $(X_t, Z_t)_{t=1,...,T}$, we compute R bootstrap versions $(Y^*_{1,r}, ..., Y^*_{T,r})_{r=1,...,R}$ via

$$Y^*_{t,r} = \hat{a}_0 + \hat{a}_1 X_t + \hat{a}_2 Z_t + \hat{a}_3 X_t Z_t + \hat{a}_4 X_t^2 + \hat{a}_5 Z_t^2 + \hat{\sigma}\epsilon_{t,r}, \tag{3.20}$$

for $t = 1, ..., T$. The error term $\epsilon_{t,r}$ is simulated using the empirical error distribution computed via the observations (Y_t, X_t, Z_t) and the parametric model previously introduced. For every version of this bootstrap (and for a given value of h, arbitrarily chosen in advanced), we estimate the conditional distribution $F_h(y_n|\mathbf{x}_k)$ using the implemented *adjusted Nadaraya-Watson* estimator giving an estimate $\tilde{F}_h(y_{n,r}|\mathbf{x}_k)^{NW}$ (simply replace the observations (Y_t, X_t, Z_t) by $(Y^*_{t,r}, X_t, Z_t)$). Here $y_{n,r}$ indicates that we estimate the conditional probability on the grid point y_n using the bootstrap r. For every single grid point y_n we now take the sample average of the absolute deviation errors between the parametric model $\hat{F}(y_n|\mathbf{x}_k)^{par}$ and the *adjusted Nadaraya-Watson* estimator $\tilde{F}_h(y_{n,r}|\mathbf{x}_k)^{NW}$ over all bootstraps and weight them using the estimate of the parametric model. The optimal bandwidth h_k for the grid point $\mathbf{x}_k$ is finally defined to be

$$h_k = \arg\min_h \Big[\sum_{n=1}^{N} \hat{F}(y_n|\mathbf{x}_k)^{par} \cdot \Big(\frac{1}{R} \sum_{r=1}^{R} \Big| \hat{F}(y_n|\mathbf{x}_k)^{par} - \tilde{F}_h(y_{n,r}|\mathbf{x}_k)^{NW} \Big| \Big) \Big]. \tag{3.21}$$

4

Portfolio Business Activities, Lévy Returns and Multivariate Stochastic Risk

Remo Crameri, Markus Leippold, Fabio Trojani

Summary. Multivariate returns of financial assets feature a number of important characteristics. First, they can jump, leading to multivariate non-normal behavior. Second, their volatilities and correlations can vary stochastically over time. Third, returns co-move with their volatilities and correlations, often negatively for equities. Fourth, returns, volatilities and correlations can co-jump, leading to self-exciting market behavior. We propose a general family of multivariate time changed Lévy processes that can simultaneously address these issues using a new class of multivariate time changes based on a matrix subordination approach. This framework includes as special cases many models in the literature, gives rise to a variety of new multivariate models and it is similarly simple to apply using the characteristic function methodology as in the univariate context.

Keywords: Multivariate time changes, multivariate subordination, Matrix jump diffusions, Wishart processes, Lévy processes

JEL Classification: C51, C52, G12, G13,

4.1 Introduction

Since the seminal work of [9], Brownian motion has been considered the benchmark process for modeling asset returns in continuous time. However, a number of important departures from the Brownian motion assumption have been identified since [9] work, both for univariate and multivariate time series of assets returns. First, returns of financial assets can contain a jump component, leading to non-normal (multivariate) returns; second, returns volatilities and correlations are not constant and vary stochastically in a persistent way; third, equity returns exhibit negative correlation with their volatilities and correlations, reflecting the well-known leverage effect. Fourth, returns, volatilities and correlations can co-jump, leading to self-exciting behavior. In the context of univariate models for returns, a number of authors has proposed important model extensions in order to parsimoniously and simultaneously capture some of these important characteristic. In particular, it has been shown in [12] that time-changed Lévy processes provide a convenient and tractable general framework to model financial returns. First, Lévy return innovations can be used to specify non Gaussian returns. Second, by running the Lévy process under a stochastic clock one can easily model stochastic risk behavior. Third, the leverage effect can be well accommodated by correlating the innovations in the Lévy process and the ones in the underlying time-change process. In this way, many well-known univariate models with stochastic risk in the literature are obtained as special cases of [12]. The usefulness of a financial model is enriched and confirmed whenever its validation and implementation is sufficiently fast. It is well-known that closed form characteristic functions and the technique of measure change are useful instruments to preserve tractability in the context of time-changed Lévy processes. In particular, [12] show that the complexity introduced by specifying the leverage effect can be neutralized trough an appropriate measure change from the risk-neutral to the so called leverage-neutral measure. Under this new (complex-valued) measure, calculations are shifted into a world where expectations can be computed as if there was no leverage: The problem of finding the characteristic function of the time-changed Lévy process in presence of leverage effects is reduced to the computation of the Laplace transform of the random time under the (complex-valued) leverage-neutral measure. Whenever the time-change is an integrated instantaneous activity rate this transform has the same form as in many well-studied problems in bond-pricing. Thus, the corresponding theory and techniques can be borrowed from the wide range of available literature.

In this paper, we focus on a class of multivariate time change procedures that are convenient for specifying multivariate stochastic risk in the context of models with multivariate Lévy returns. Multivariate models for returns featuring multiple stochastic risks are well-established in the econometrics literature; see, e.g., [8] for a review on the multivariate GARCH approach. Recently, a new class of continuous-time models for multivariate stochastic risk have been proposed by a number of authors, by directly specifying stochastic processes for symmetric positive-definite covariance matrices. [7] introduce matrix-valued positive definite Ornstein-Uhlenbeck processes, while [20] propose a family of matrix affine jumps diffusions (AJD) with finite activity as a convenient framework for specifying multivariate risks in finance. Applications of these settings to different problems in finance are given in [16], [10], [14], [17] and [24].

The extension of [12] approach to a general and tractable multivariate setting with multiple sources of stochastic risk, multivariate leverage effects and general stochastic dependence between returns is difficult. A major issue is that in order to properly model multivariate stochastic risks with Lévy processes one needs an appropriate multivariate time change procedure, i.e., different time changes, or subordinators, for different assets, which has to be consistent with the properties of a stochastic covariance matrix of returns, like, e.g., symmetry and positive definiteness.[1] These features pose serious challenges for the extension to multivariate time-change techniques and do not arise in the one-dimensional case, in which Heston-type volatility models can always be rephrased as time-changed Brownian motion using as a time-change process the integrated variance; see, e.g., [3].

Our paper introduces a multivariate subordination methodology that is convenient to specify a family of dependent time-changes that can be consistent with the properties of a stochastic covariance matrix of returns. In our approach, multivariate time-changes are defined based on increasing processes of positive definite and symmetric matrices. To the best of our knowledge, this is the first approach in the literature presenting this idea. In this way, we extend in a natural way the general approach in [12] to a potentially broad multivariate setting with multiple sources of stochastic risk, multivariate leverage effects and stochastic dependence between returns. The more specific contributions to the literature are the following. First, we can specify multivariate Lévy processes with stochastic

[1] Multivariate time changes appeared recently in the literature. Independent multivariate subordinators have been considered, among others, in [4] and [13]. [21] introduce a single-factor multivariate time change that implies a constant correlation between returns of different assets.

dependence between returns, in contrast to the constant correlation implied by previous multivariate time change procedures; see, e.g., again [21]. Second, we derive natural extensions of univariate subordinated Lévy processes to their matrix-valued subordinated processes. For instance, we introduce multivariate counterparts of well-know univariate models, including the VG model in [23] and [22] and the NIG model in [2]. Third, we can account for multivariate leverage effects by correlating the multivariate shocks in returns and our multivariate time changes. In this context, we derive the relevant expressions for the leverage-neutral measure arising under our approach. Fourth, we show how to specify our multivariate time-changes in order to preserve an affine structure and closed form transform expressions under the (multivariate) leverage neutral measure.

The paper is organized as follows. Section 4.2 introduces the framework and defines our multivariate time-changes. In a second step, the leverage-neutral probability measure is derived. In Section 4.3, we show how one can use our methodology for generating new classes of multivariate Lévy processes. Section 4.4 defines portfolio time-changes based on matrix AJD, for which the Laplace transform under the leverage-neutral measure is derived in closed form. Section 4.5 concludes.

4.2 Multivariate Financial Modeling Using Families of Lévy Processes

The model framework is presented in this section. For brevity of exposition and simplicity of notation, we consider a market with two price processes $(S_{1t}, S_{2t})_{t\geq 0}$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with a standard complete filtration $\mathbf{F} := (\mathcal{F}_t)_{t\geq 0}$, which is generated by a Markov process $X := (X_t)_{t\geq 0}$, i.e., $\mathcal{F}_t = \sigma(X_t)$. The extension of our approach to higher dimensions can be carried out at the cost of notational complexity. The bivariate log return process is defined by $R_t := (R_{1t}, R_{2t}) := (\log(S_{1t}/S_{10}), \log(S_{2t}/S_{20}))$. We denote by $u'R_t := u_1 R_{1t} + u_2 R_{2t}$, where $u \in \mathbb{R}^2$, any linear combination of log returns with weights $u = (u_1, u_2)'$.

In order to develop a multivariate model for returns based on time-changed Lévy processes, we need to specify a family of well-defined conditional transition densities for bivariate return process $(R_t)_{t\geq 0}$. [12] model asset prices as exponential affine functions of a given vector of Lévy shocks, time-changed by a fixed business time. We propose to extend this approach by specifying a parametric family of time-changed Lévy processes, parameterized by $u \in \mathbb{R}^2$, that models the multivariate features of the underlying stochastic risk structure. To this end, we specify the joint distribution of $(R_t)_{t\geq 0}$ indirectly, by modeling the distribution of any process $(u'R_t)_{t\geq 0}$ for arbitrary $u \in \mathbb{R}^2$.

4.2.1 The Basic Approach

The main insight of our approach is to specify a family of time-changed Lévy processes, parameterized by $u \in \mathbb{R}^2$, modeling the stochastic properties of process $\{(u'R_t)_{t\geq 0} : u \in \mathbb{R}^2\}$, in a way that at the same time ensures a family of well-defined joint conditional distributions for bivariate return process $(R_t)_{t\geq 0}$. We focus on bivariate stationary Markov processes, implying a family of conditional characteristic functions defined by:

$$\Phi_{R_1,R_2}(u,t,t+\Delta,x) := \mathbb{E}\left[\exp(iu'R_{t+\Delta})\Big|X_t = x\right] = \mathbb{E}\left[\exp(iu'R_{\Delta})\Big|X_0 = x\right] \quad (4.1)$$

$$=: \Phi_{R_1,R_2}(u,\Delta,x) \quad , \quad (4.2)$$

where the equality follows from the stationarity assumption. The next assumption introduces our family of time-changed Lévy processes for modeling multivariate returns.

Assumption 4.1. *Let processes $(R_t(u))_{t\geq 0}$, $u \in \mathbb{R}^2$, be defined by:*

$$R_t(u) := \sum_{i=1}^{d} \theta_i L^u_{iT^u_{it}} := \theta' L^u_{T^u_t} \quad (4.3)$$

where $\theta \in \mathbb{R}^d$, $\{L^u_t : u \in \mathbb{R}^2\}$ is a parametric family of $d-$dimensional Lévy processes, identically distributed as L, and $\mathcal{T} := \{T^u_t := (T^u_{1t},\ldots,T^u_{dt})' : u \in \mathbb{R}^2\}$ is a suitable family of $d-$dimensional time-changes parameterized by $u \in \mathbb{R}^2$. We denote by

$$\Psi_L(\theta) = -i\mu'\theta + \frac{1}{2}\theta'\Sigma\theta + \int_{\mathbb{R}^d_0}\left(1 - e^{i\theta'x} + i\theta'x\mathbf{1}_{|x|<1}\right)\Pi_L(dx) \;, \quad (4.4)$$

the characteristic exponent of L.

Consider the function:

$$u \mapsto \Phi(u,\Delta,x) := E\left[\exp(iR_\Delta(u))\Big|X_0 = x\right] \;. \quad (4.5)$$

Under appropriate conditions on time-changed process $L^u_{T^u_t}$, $\Phi(\cdot,\Delta,x)$ can be used to specify a well-defined family of conditional probability densities $\Phi_{R_1,R_2}(u,\Delta,x)$ for bivariate return process $(R_t)_{t\geq 0}$. Moreover, under straightforward assumptions, $\Phi(u,\Delta,x)$ can be written as the Laplace transform of time change process $(T^u_t)_{t\geq 0}$.

Lemma 4.1. *Let $(L_t^u)_{t\geq 0}$ and $(T_t^u)_{t\geq 0}$ be independent processes. If either (i) L_t^u has independent components, or (ii) T_t^u has identical components, then:*

$$\Phi(u,\Delta,x) = \mathbb{E}\left[\exp(i\theta' L^u_{T^u_\Delta})|X_0 = x\right] = \mathbb{E}\left[\exp\left(-\sum_{i=1}^{d}\Psi_{L_i}(\theta_i)T^u_{i\Delta}\right)\Big|X_0 = x\right] \quad (4.6)$$

$$= \mathcal{L}_{T^u}(-\Theta_L(\theta),\Delta,x)\ , \quad (4.7)$$

with $\mathcal{L}_{T^u}(\cdot,\Delta,x)$ the conditional characteristic function of T^u_Δ and $\Theta_L(\theta) := (\Psi_{L_1}(\theta_1),\ldots,\Psi_{L_d}(\theta_d))'$.

Equation (4.6) makes explicit that the dependence of $\Phi(u,\Delta,x)$ on u is only via the parametric dependence of T^u on u:

$$\Phi(\cdot,\Delta,x) : u \longmapsto T^u \longmapsto \mathcal{L}_{T^u}(-\Theta_L(\theta),\Delta,x)\ .$$

This simple remark highlights that the multivariate dependence properties implied by our modeling approach are completely determined by the specification of the family of multivariate time-changes $\{(T_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$.

4.2.2 Portfolio Time Changes and Matrix Markov Processes

The new insight of our approach derives from the family of time-changes T_t^u in equation (4.3). We call such families portfolio time-changes.

Definition 4.1. *A portfolio time-change is a family $\mathcal{T} := \{(T_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ of $d-$dimensional time change processes $(T_t^u)_{t\geq 0}$ on probability space $(\Omega,\mathcal{F},\mathbb{P})$, i.e., such that for any u:*

1. *Process $(T_{it}^u)_{t\geq 0}$, $i = 1,\ldots,d$, is positive, increasing, and right continuous with left limits, satisfying the usual regularity conditions,*
2. *For any $t \geq 0$ and $i = 1,\ldots.n$, $\{T_{it}^u \leq t\}$ is $\mathcal{F}_t-$ measurable,*
3. *$T_{it}^u \to \infty$ almost surely, as $t \to \infty$, for all $i = 1,\ldots,d$.*

Essentially, in order to generate well-defined portfolio time-changes, we need to introduce parametric families of increasing positive semi-martingales. A convenient and systematic way of achieving this task is by means of families of 2×2 matrix-valued positive definite and increasing Markov processes generating filtration $\{\mathcal{F}_t : t \geq 0\}$.[2]

[2] Examples of such processes are matrix subordinators. Finite activity compound Poisson-type matrix subordinators can be easily constructed using any finite probability distribution on the cone $\mathcal{S}_n^+$ of

Assumption 4.2. $\{(X_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ *is a family of symmetric and positive definite* 2×2 *matrix-valued processes adapted to* $(\mathcal{F}_t)_{t\geq 0}$ *such that:*

1. $(X_t^u)_{t\geq 0}$ *is increasing, and right continuous with left limits, satisfying the usual regularity conditions,*
2. $tr(X_t^u)^u \to \infty$ *as* $t \to \infty$.

Condition 2. ensures that any quadratic for in X_t^u converges to ∞ as $t \to \infty$.

The next example illustrates a useful way of specifying portfolio time-changes based on Assumption 4.2.

Example 4.1. Let V_t^u be the unique symmetric positive definite square root of process X_t^u in Assumption 4.2 and define $T_{it}^u := tr(uu^{'}(V_t^{ui})(V_t^{ui})^{'})$, where V_t^{ui} is the i-th column of matrix V_t^u. By construction, $\{T_t^u := (T_{1t}^u, T_{2t}^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ is a portfolio time-change such that $T_{1t}^u + T_{2t}^u = tr(uu^{'} X_t^u)$. Under the conditions of Lemma 4.1, this way of specifying T_t^u allows for a direct computation of function $\Phi(u, \Delta, x)$ using the conditional Laplace transform $\mathcal{L}_X(\cdot, \Delta, x)$ of X_Δ. Indeed, by choosing L_t^u such that $\Psi_{L_1} = \Psi_{L_2}$ and letting $R_t(u) = L^u_{1T^u_{1t}} + L^u_{2T^u_{2t}}$, then:

$$\Phi(u, \Delta, x) = \mathbb{E}\left[\exp(-\Psi_{L_1}(1) tr(uu' X_\Delta) | X_0 = x\right] = \mathcal{L}_X(-\Gamma, \Delta, x) \;, \tag{4.8}$$

where $\Gamma = \Psi_{L_1}(1)uu'$.

4.2.3 Portfolio Time-Changed Brownian Motion

In principle, the family of Lévy processes $\{(L_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ can be arbitrarily chosen. Based on Monroe's theorem and the Fundamental Theorem of Asset Pricing, a large class of arbitrage-free continuous-time models can be specified using a dynamics implied by time-changed Brownian motion. Moreover, many univariate Lévy processes broadly used in financial modeling are derived from a time-changed Brownian motion.[3] As a consequence, we consider in more detail in this section the Brownian motion setting.

symmetric positiv definite matrices for the jump size; see e.g., [18], for some concrete examples. [5] and [6] introduce matrix subordinators that generalize stable, tempered stable, Gamma and Inverse Gaussian subordinators, and which can allow for high-frequency matrix-valued jumps.

[3] Based on the Fundamental Theorem of Asset Pricing, price processes are semimartingales. Monroe's theorem states that every semimartingale Z_t can be rewritten as a time-changed Brownian motion B_{T_t} for a family of stopping times T_t defined on a suitably extended probability space. For example, the

Assumption 4.3. $\{(B_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ *denotes a parametric class of Brownian motions in* $\mathbb{R}^d$ *indexed by* u.

In order for function $\Phi(u, \Delta, x)$ in equation (4.5) to define a well-defined family of conditional characteristic functions for a bivariate return process, i.e.,

$$\Phi(u, \Delta, x) = \Phi_{R_1,R_2}(u, \Delta, x) \ ,$$

for some bivariate return distributions, two conditions have to be satisfied. First, the Chapman-Kolmogoroff equation has to hold:

$$\Phi(u, \Delta_1 + \Delta_2, x) = E\left[\Phi(u, \Delta_2, X_{\Delta_1}^u)\Big| X_0^u = x\right] \ . \tag{4.9}$$

Second, $\Phi(\cdot, \Delta, x)$ has to be the characteristic function of a bivariate random variable.

Proposition 4.1. *Let the hypotheses of Lemma 4.1 and Assumption 4.2 be satisfied. (i) If portfolio time-change* $\{(T_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ *and* $R_t(u)$ *are specified as in Example 4.1, then the Chapman-Kolmogoroff equation (4.9) is satisfied if and only if* $(X_t)_{t\geq 0}$ *is a Markov process, i.e., if for* $\Gamma \in \mathbb{C}^{2\times 2}$ *the following identity holds:*

$$\mathcal{L}_X(\Gamma, \Delta_1 + \Delta_2, x) = \mathbb{E}\left[\mathcal{L}_X(\Gamma, \Delta_2, X_{\Delta_1})|X_0 = x\right]. \tag{4.10}$$

(ii) If additionally Assumption 4.3 is satisfied, then function $\Phi(u, \Delta, x)$ *specifies a well-defined family of bivariate conditional densities for return process* $(R_t)_{t\geq 0}$.

Proof. The first statement follows from the definition of $R_t(u)$ under Assumption 4.2 in the context of Example 4.1 and Lemma 4.1. The second statement follows from the properties of the multivariate characteristic function of a Gaussian variable, since $(\Psi_{L_1}, \Psi_{L_2}) = (1/2, 1/2)'$. By iterated expectations, we obtain:

$$E[\exp(R_\Delta(u))|X_0 = x] = \mathcal{L}_X(\frac{1}{2}uu', \Delta, x) = E[\exp(u'X_\Delta u/2)|X_0 = x], \tag{4.11}$$

which is the integrated characteristic function (with respect to the conditional density of X_Δ) of a bivariate zero mean Gaussian variable with covariance matrix X_Δ. This concludes the proof. □

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univariate Variance Gamma model of [23] and [22] and the Normal Inverse Gaussian model of [2] are obtained by subordinating Brownian motion with univariate Gamma and Inverse Gaussian processes, respectively.

4.2.4 Portfolio Business Activities

Increasing matrix processes in Assumption 4.2 can be specified either directly as a matrix subordinator, i.e., a matrix Lévy process with positive definite increments, or as a finite variation matrix process obtained, e.g., by integrating another positive definite matrix process. In the latter case, we obtain a systematic way of specifying portfolio business times with a given local activity, which can follow a state process featuring time dependence. An example of a matrix subordinator is the Ornstein-Uhlenbeck-type matrix process introduced in [7].[4] The matrix AJD setting introduced in the literature by [20] are examples of matrix processes that can be naturally used to specify a variety of portfolio business activities with time series dependence.

Example 4.2. Let $\{(\Sigma_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ be a family of positive definite matrix processes and define:

$$T_{it}^u = tr\left(uu'\int_0^t V_s^{ui}V_s^{ui'}ds\right) \ , \tag{4.13}$$

where V_t^u is the square root of Σ_t^u. Then, (T_{1t}^u, T_{2t}^u) defines a portfolio time-change of bounded variation with local business activities given by:

$$tr\left(uu'V_t^{ui}V_t^{ui'}\right) \quad , \quad i = 1,2 \quad . \tag{4.14}$$

Therefore, $\{(T_{1t}^u, T_{2t}^u)_{t\geq 0} : u \in \mathbb{R}^2\}$, can be used to model the stochastic multivariate risk of $R_t(u)$, as:

$$Var_t(dR_t(u)) = tr\left(uu'(V_t^{u1}V_t^{u1'} + V_t^{u2}V_t^{u2'})dt\right) = tr(uu'\Sigma_t^u)dt \ . \tag{4.15}$$

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[4] Let $\{(J_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ be a family of matrix valued Lévy subordinators, with $E[\ln^+ \parallel J_t^u \parallel] < \infty$, and M be a fix 2×2 matrix of parameters such that $\sigma(M) \subset (-\infty, 0) + i\mathbb{R}$. An Ornstein-Uhlenbeck-type process X_t^u is defined as the solution of the stochastic differential equation:

$$dX_t^u = (MX_t^u + X_t^u M')dt + dJ_t^u, \tag{4.12}$$

with initial condition $X_0^u = x \in \mathcal{S}_n^+$. Technical conditions for existence and uniqueness of an infinitely divisible stationary solution of (4.12) are provided in [7]. As in the univariate case, depending on the choice of the Lévy characteristics of the subordinators J_t^u, different types of positive semi-definite Ornstein-Uhlenbeck type processes are obtained. Second order moment structure and close form solution for the integrated process can be found in [1].

The previous example suggests that a number of multivariate models with stochastic risk can be embedded into our modeling approach, using portfolio business activities. The next example shows that using portfolio time-changed Brownian motions, we can encompass (at least) a large class of multivariate Heston-type stochastic volatility models.

Example 4.3. Let $B_t = (B_{1t}, B_{2t})'$ be a bivariate standard Brownian motion and consider the following Heston-type multivariate stochastic volatility model for returns:

$$\begin{pmatrix} dS_{1t} \\ dS_{2t} \end{pmatrix} = \begin{pmatrix} S_{1t}(V_t^{11} dB_{1t} + V_t^{12} dB_{2t}) \\ S_{2t}(V_t^{21} dB_{1t} + V_t^{22} dB_{2t}) \end{pmatrix} \tag{4.16}$$

where

$$V_t := \begin{pmatrix} V_t^{11} & V_t^{12} \\ V_t^{21} & V_t^{22} \end{pmatrix}$$

is driven by a 2×2 symmetric matrix diffusion process independent of Brownian motion B and $\Sigma_t := V_t V_t'$. Denote by e_i the $i-$th unit vector. Given this specification, we have:

$$u' R_t = u' \int_0^t V_s dB_s - \frac{1}{2} \int_0^t (u_1 \Sigma_s^{11} + u_2 \Sigma_s^{22}) ds \ ,$$

and:

$$\begin{aligned} \Phi_{R_{1t},R_{2t}}(-iu) &= \mathbb{E}\left[\exp(u' R_t)\right] \\ &= \mathbb{E}\left[\exp\left(\frac{1}{2} tr(uu' \int_0^t \Sigma_s ds) - \frac{1}{2} tr\left((u_1 e_1 e_1' + u_2 e_2 e_2') \int_0^t \Sigma_s ds\right)\right)\right] \\ &= \mathbb{E}\left[\exp\left(\frac{1}{2} tr\left((uu' - u_1 e_1 e_1' - u_2 e_2 e_2') \int_0^t \Sigma_s ds\right)\right)\right] \ . \end{aligned} \tag{4.17}$$

Let now $\{(V_t^u)_{t \geq 0} : u \in \mathbb{R}^n\}$ be a family of matrix processes distributed as $(V_t)_{t \geq 0}$ and define for $k = 1, 2$ the portfolio time changes:

$$T_{kt}^u = tr(uu' \int_0^t V_s^{uk} V_s^{uk'} ds) \tag{4.18}$$

where V_t^{uk} is the $k-$th column of V_t^u. In particular, note that $T_{1t}^u + T_{2t}^u = tr(uu' \int_0^t \Sigma_s ds)$. Consider now the following time-changed process for the log return portfolio $R_t(u)$:

$$R_t(u) = B_{1T_{1t}^u}^u + B_{2T_{2t}^u}^u - \frac{1}{2}(u_1(T_{1t}^{e_1} + T_{2t}^{e_1}) + u_2(T_{1t}^{e_2} + T_{2t}^{e_2})) \ . \tag{4.19}$$

where $\{(B_t^u)_{t \geq 0} : u \in \mathbb{R}^2\}$ is a family of bivariate standard Brownian motions independent of $\{(V_t^u)_{t \geq 0} : u \in \mathbb{R}^2\}$. With specification (4.19), we obtain:

$$\begin{aligned} \Phi_{R_t(u)}(-iu) &= \mathbb{E}\left[\exp\left(\frac{1}{2}(T_{1t}^u + T_{2t}^u) - \frac{1}{2} tr\left((u_1 e_1 e_1' + u_2 e_2 e_2') \int_0^t \Sigma_s ds\right)\right)\right] \\ &= \mathbb{E}\left[\exp\left(\frac{1}{2} tr\left((uu' - u_1 e_1 e_1' - u_2 e_2 e_2') \int_0^t \Sigma_s ds\right)\right)\right] \ , \end{aligned} \tag{4.20}$$

because $T^u_{1t} + T^u_{2t} = tr(uu' \int_0^t \Sigma^u_s ds)$. Since $\Phi_{R_t(u)}(-iu)$ coincides with the Laplace transform of R_t under multivariate Heston-type model (4.16) (given in (4.17)), these models are nested by our modeling approach (4.3) based on families of time-changed Lévy processes.

4.2.5 Leverage Effect and Complex Leverage-Neutral Measure

Equation (4.6) in Lemma 4.1 holds provided that L^u_t and T^u_t are independent. In this context, the computation of the characteristic function of returns is feasible analytically as soon as the characteristic function $\mathcal{L}_{T^u}(\cdot, \Delta, x)$ is tractable. In order to easily capture the well-known leverage effect in our multivariate approach, it is necessary to consider settings in which Lévy processes L^u_t and portfolio time change T^u_t can be correlated.

Under specification (4.3), the computation of function $\Phi(u, \Delta, x)$ depends on two sources of randomness, the first linked to Lévy process L^u_t and the second linked to the stochastic time T^u_t. Using the concept of a leverage-neutral measure, [12] show that it is possible to simplify the computation of the characteristic function of returns by a simple bond price-type formula, also when Lévy shocks and the underlying business times are correlated. We apply this idea to our multivariate time-change approach and show how the computation of function $\Phi(u, \Delta, x)$ is similarly easily performed as when L^u_t and T^u_t are independent. This is done by using a family of leverage-neutral measures $\{\mathbb{Q}(\theta, u) : u \in \mathbb{R}^2\}$ that incorporates the dependence of the leverage effect on $u \in \mathbb{R}^2$.

Theorem 4.1. *Assume that* $\{(T^u_t)_{t\geq 0} : u \in \mathbb{R}^2\}$ *is a portfolio time change of bounded variation. Let either (i)* L^u_t *have independent components or (ii)* $T^u_{1t} = \ldots = T^u_{dt}$. *Then, under model (4.3), function* $\Phi(u, \Delta, x)$ *in equation (4.6) is given by the Laplace transform of portfolio time change* T^u_Δ, *under a complex-valued portfolio-dependent leverage neutral measure* $\mathbb{Q}(\theta, \cdot) : u \mapsto \mathbb{Q}(\theta, u)$, *evaluated in* $\Theta_L := \Theta_L(\theta)$:

$$\Phi(u, \Delta, x) = \mathbb{E}^{\mathbb{Q}(\theta,u)}\left[\exp\left(-\Theta_L' T^u_\Delta\right)\right] =: \mathcal{L}^{\mathbb{Q}(\theta,u)}_{T^u}(-\Theta_L, \Delta, x) \ , \tag{4.21}$$

where complex-valued measure $\mathbb{Q}(\theta, u)$ *has density* $M(\theta, u)$ *with respect to* $\mathbb{P}$ *given by:*

$$M_t(\theta, u) := \left.\frac{d\mathbb{Q}(\theta, u)}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \exp\left[i\theta' L^u_{T^u_t} + \Theta_L' T^u_t\right]. \tag{4.22}$$

The expectation in equation (4.21) is the conditional Laplace transform of vector T^u_Δ under the new measure $\mathbb{Q}(u, \Theta)$, evaluated at $-\Theta_L$. Equation (4.21) is similar to the expression in the leverage-free case derived in Equation (4.6), with the only difference that the expectation is taken under the complex-valued measure $\mathbb{Q}(\theta, u)$. The proof of Theorem 4.1 proceeds in two steps. First, we show that the process $M_t(\theta, u)$ is a well-defined complex-valued martingale under the original measure $\mathbb{P}$, with respect to the filtration generated by processes $\{(L_t^u, T_t^u)\}$. Second, we derive Equation (4.21).

Lemma 4.2. *Let the conditions of Theorem 4.1 be satisfied. Then process* $M_t(\theta, u) := \exp\left[i\theta' L^u_{T^u_t} + \Theta_L' T^u_t\right]$ *is a well-defined complex-valued* $\mathbb{P}-$*martingale with respect to the filtration* $\mathcal{F}^u_t$ *generated by processes* $\{(L_t^u, T_t^u)\}$.

Proof. Processes $Y^u_{kt} := \exp\left[i\theta_k L^u_t + \Theta_{kL} t\right]$, $k = 1, \ldots, d$ are independent Wald martingales such that $M_t(\theta, u) = \prod_{i=k}^d Y^u_{kT^u_{kt}}$ is a martingale as well. By the optional stopping theorem, replacing in Y^u_{kt} the calendar time t with a locally deterministic time change T^u_{kt}, $k = 1, ..., d$, does not alter the martingale behavior and the conditional orthogonality of these processes. Hence, density process $M_t(\theta, u)$, which is the product of orthogonal martingales $Y^u_{kT^u_{kt}}$, is again a (complex-valued) martingale with respect to the filtration $\mathcal{F}^u_t$. □

Using Lemma 4.2, equation (4.21) in Theorem 4.1 follows immediately:

$$\begin{aligned}
\Phi(u, \Delta, x) &:= \mathbb{E}\left[\exp\left(i\theta' L^u_{T^u_\Delta}\right)\Big| X^u_0 = x\right] \\
&= \mathbb{E}\left[M_\Delta(\theta, u)\exp\left(-\Theta_L' T^u_\Delta\right)\Big| X^u_0 = x\right] \\
&= \mathbb{E}^{\mathbb{Q}(\theta,u)}\left[\exp\left(-\Theta_L' T^u_\Delta\right)|X^u_0 = x\right] =: \mathcal{L}^{\mathbb{Q}(\theta,u)}_{T^u_\Delta}(-\Theta_L, \Delta, x)\ .
\end{aligned}$$

A measure change from the risk-neutral to the so called leverage-neutral measure removes the computational difficulties due to the leverage effect: The leverage effect is embedded into leverage-neutral measure $\mathbb{Q}(\theta, u)$ and expectations can be performed as if there was no leverage. Thus, the characteristic function of the time-changed Lévy process is the conditional Laplace transform of the random time under the leverage-neutral measure, evaluated at $-\Theta_L$. Moreover, in cases where portfolio time change T^u_t is generated by a portfolio business activity driven by a matrix process, as in Example 4.2, Theorem 4.1 provides a potentially powerful tool for computing function $\Phi(u, \Delta, x)$ by a simple bond price-type formula:

$$\begin{aligned}
\Phi(u, \Delta, x) &= \mathbb{E}^{\mathbb{Q}(\theta^\star,u)}\left[\exp\Big(-\Psi_{L^1}(1)(T^u_{1t} + T^u_{2t})\Big)\Big| X^u_0 = x\right] &(4.23)\\
&= \mathbb{E}^{\mathbb{Q}(\theta^\star,u)}\left[\exp\left(-tr\Big(\Psi_{L^1}(1)uu'\int_0^t X^u_s ds\Big)\right)\Big| X^u_0 = x\right]\ , &(4.24)
\end{aligned}$$

where $\theta^\star = (1, 1)'$. Interpreting process $r^u_t := tr(\Psi_{L^1}(1)uu'X^u_s)$ as a short interest rate process, Equation (4.24) provides a formula similar to the pricing formula of a zero coupon bond, parameterized by $u \in \mathbb{R}^2$. At the same time, $\Phi(u, \Delta, x)$ can be interpreted as the Laplace transform of the integrated matrix process $\int_0^t X_s^u ds$, under the leverage-neutral measure $\mathbb{Q}(\theta^\star, u)$, evaluated in $-\Psi_L^1(1)uu'$. We can therefore borrow existing techniques for the computation of $\Phi(u, \Delta, x)$ and make closed-form expressions for this function readily available when the leverage-neutral Laplace transform of $\int_0^t X_s^u ds$ can be computed analytically. An important class of matrix processes for which these computations can be carried out explicitly is the family of matrix AJD, introduced in the literature by [20].

The concrete specification analysis of tractable portfolio time changes $\{(T_t^u)_{t\geq 0} : u \in \mathbb{R}^2\}$ and corresponding families of time-changed Lévy processes within our framework is addressed in Section 4.4.

4.3 Multivariate Lévy Returns Generated by Matrix Subordination

Subordination of univariate Brownian motion by an independent univariate Lévy process has been extensively studied. [15], among others, show that a number of well-known Lévy processes can be written as time-changed Brownian motion, where the subordinating Lévy process is specified using a specific infinitely divisible distribution. The Normal Inverse Gaussian (NIG) and Variance Gamma (VG) models, e.g., are obtained by subordinating Brownian motion with univariate Inverse Gaussian and Gamma processes, respectively. Extensions of univariate subordinators, like Inverse Gaussian and Gamma subordinators, to their matrix-valued counterparts have been recently proposed in a number of papers; see, e.g., [5], [6] and [25].[5] It is well-known that a Lévy process subordinated by a Lévy subordinator is again a Lévy process. In this section, we show how our multivariate time-change approach combined with a matrix subordination can be used to generate a new class of multivariate Lévy processes, featuring known margins and new dependence features between the single components of the process.

[5] A key issue in the construction of matrix subordinators is the specification of a suitable infinitely divisible matrix distribution. For example, the usual matrix variate extension of the Gamma density (see e.g. [18]) is not infinitely divisible and one needs to slightly change its definition in order to obtain a well-defined matrix subordinator. As is the case for standard infinitely divisible distributions in $\mathbb{R}^n$, infinitely divisible matrix distribution are characterized by their Lévy-Khinchin representation; see, for instance, [6].

4.3.1 Multivariate Lévy Returns as Portfolio Time-Changed Brownian Motions

Assumption 4.4. *$B_t^u := (B_{1t}^u, B_{2t}^u)$ is a family of standard Brownian motions in $\mathbb{R}^2$ indexed by $u \in \mathbb{R}^2$. X_t^u is a family of matrix subordinators, identically distributed according to an infinitely divisible random variable X, indexed by $u \in \mathbb{R}^2$ and independent of B_t^u. $\mathcal{K}_X$ denotes the cumulant transform of X. Let $T_{it}^u := tr(uu^{'}(V_t^{ui})(V_t^{ui})^{'})$, where V_t^u is the unique symmetric positive definite square root of X_t^u. $R_t(u)$ is specified as the portfolio time-changed Brownian motion:*

$$R_t(u) = B_{1T_{1t}^u}^u + B_{2T_{2t}^u}^u \tag{4.25}$$

By construction, $R_t(u)$ defines a family of univariate Lévy process indexed by $u \in \mathbb{R}^2$, which are obtained by subordinating a Lévy process with a quadratic form of a matrix subordinator. It follows that $\Phi(u, \Delta, x) := E[\exp(iR_\Delta(u))|X_0^u = x] = \Phi(u, \Delta)$, given the iid structure of the matrix subordinator, where:

$$\Phi(u, \Delta) = \mathbb{E}\left[\exp\left(\frac{1}{2}tr(uu^{'}X_\Delta)\right)\right] = \mathcal{L}_{X_\Delta}\left(\frac{1}{2}uu^{'}\right) = \exp\left(-\Delta\mathcal{K}_X\left(\frac{1}{2}uu^{'}\right)\right). \tag{4.26}$$

This is the general expression for the Laplace transform of a new class of bivariate Lévy return processes. This expression is tractable analytically whenever the Laplace transform of X is known in closed-form. The marginal distributions of the model in equation (4.26) are characterized by evaluating $\Phi(\cdot, \Delta)$ in the two unit vectors $e_1 = (1,0)'$ and $e_2 = (0,1)'$:

$$\Phi(e_1, \Delta) = \exp\left(-\Delta\mathcal{K}_X\left(\frac{1}{2}e_1e_1^{'}\right)\right) \ , \ \Phi(e_2, \Delta) = \exp\left(-\Delta\mathcal{K}_X\left(\frac{1}{2}e_2e_2^{'}\right)\right) \ . \tag{4.27}$$

From these expressions, we see that the marginal distributions implied for $(R_t(e_1)_{t\geq 0})$ and $(R_t(e_2)_{t\geq 0})$ are those of a univariate time-changed Brownian motion, with time-changes $T_{1t} := X_t^{11}$ and $T_{2t} := X_t^{22}$ given by the diagonal elements of matrix subordinator X_t.[6]

In the sequel, we consider in more detail concrete examples of new multivariate Lévy processes using our approach, based on matrix-valued extensions of well-known univariate

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[6] Specifying the diagonal elements of a matrix process X_t as univariate subordinators and setting the off-diagonal elements to zero leads to a simple example of a matrix subordinator, called diagonal matrix subordinator, see [25]). In the bivariate case, e.g, a Gamma process can be used as the first diagonal element and an independent Inverse Gaussian process as the second diagonal element. By construction, in this case $R_t(e_1)$ is a variance gamma process, whereas $R_t(e_2)$ is an independent normal inverse gaussian process.

subordinators, like the Gamma, the Inverse Gaussian and the Compound Poisson processes with positive jumps. Other examples of matrix-subordinators that could be applied in the context of our multivariate approach include Bessel matrix-subordinators ([6]) and matrix subordinators defined by the quadratic variation of multidimensional Lévy processes ([25]).

4.3.2 Gamma-type, Tempered and Inverse Gaussian Matrix Subordinators

[6] develop Gamma-type and simple tempered matrix distributions that are infinitely divisible. The corresponding multivariate Lévy density is given by

$$h_\beta(X,\Sigma) := \frac{|\Sigma|^{-(n+1)/2}\exp(-tr(X\Sigma^{-1}))}{tr(X\Sigma^{-1})^{n(n+1)/2+\beta}}, \tag{4.28}$$

where $\Sigma \in \mathcal{S}_n^+$ and $0 \leq \beta < 1$. We write $X \sim G_\beta(\Sigma)$ to indicate that the random matrix X has the distribution associated to Lévy density (4.28). $\beta = 0$ corresponds to the Gamma-type distribution: In this case, $tr(X)$ follows a Gamma distribution when $\Sigma = I_n$, and density (4.28) is the natural (infinitely divisible) extension of the univariate Lévy density of a gamma process with unit mean and scaling coefficient σ; see, e.g., Revuz and Yor (1991, p. 110). For $0 < \beta < 1$ and $\Sigma = I_n$, equation (4.28) parameterizes the family of β-tempered matrix distributions, in which the special case $\beta = 1/2$ is the (infinitely divisible) matrix extension of the univariate Inverse Gaussian distribution; see [5].

Lemma 4.3. *([6]) By changing to the polar decomposition $X = rV$, $r = tr(X)$, for which we have that $tr(V) = 1$, one has:*

1. *For $X \sim G_0(\Sigma)$: $\mathcal{K}_X(\Theta) = \int\limits_{\boldsymbol{S}^n \cap \mathcal{S}_n^+} \ln(1 + tr(V\Sigma\Theta))^{-1} dV$,*
2. *For $X \sim G_\beta(I_n)$, $0 < \beta < 1$: $\mathcal{K}_X(\Theta) = -k_\beta \int\limits_{\boldsymbol{S}^n \cap \mathcal{S}_n^+} \ln(1 + tr(V\Theta))^\beta dV - c_n$,*

where $k_\beta := \Gamma(1-\beta)/\beta$, $c_n := \frac{\pi^{n/2}}{[n(n+1)/2-1]!}$ and $\boldsymbol{S}^n \cap \mathcal{S}_n^+$ is the intersection of the unit sphere of dimension $n \times n$ with the positive definite cone.

Lemma 4.3 implies that when $X \sim G_0(\Sigma)$ random variable $tr(\Sigma X)$ follows a one-dimensional gamma convolution. Generalized gamma convolutions build an important class of infinitely divisible distributions, firstly introduced by Thorin in 1977. Their characteristic function takes the form

$$\Phi(s) = \exp(\int_0^\infty \ln(1 + s/t))\mu(dt), \tag{4.29}$$

where $\mu(dt)$ is a nonnegative measure on $(0,\infty)$, called Thorin measure. The univariate Gamma distribution arises as a special case in this class when $\mu(dt)$ is the Dirac measure at 1. When $X \sim G_0(I_n)$ and $0 < \beta < 1$ the resulting matrix law is related to the one-dimensional tempered β-stable distributions, introduced in [27], because any one-dimensional marginal of X follows a tempered β-stable distribution.

4.3.2.1 Marginal Distributions: Gamma Subordinators

Using Lemma 4.3, we can characterize more concretely the specific form of function (4.26) implied by different matrix subordinators. We start with Gamma-type matrix subordinators.

Corollary 4.1. *Let $(X_t)_{t\geq 0}$ be a Gamma-type matrix subordinator, i.e. $X \sim G_0(\Sigma)$. It then follows:*

$$-\frac{1}{\Delta}\ln \Phi(u,\Delta) = \int_{\mathcal{S}^n\cap\mathcal{S}_n^+} \ln\left(1+\frac{1}{2}tr(uu'\Sigma V)\right)^{-1} dV = \tag{4.30}$$

$$\int_0^\infty \ln\left(1+\frac{1}{2}v\right)^{-1} \nu_{uu'\Sigma}(dv) \tag{4.31}$$

where $\nu_{uu'\Sigma}(B) := \int_{\mathcal{S}^n\cap\mathcal{S}_n^+} I_B(tr(uu^{'}\Sigma V))\nu_V(dV)$ is the measure induced on $(0,\infty)$ trough the transformation $V \to tr(uu^{'}\Sigma V)$.

The last expression in equation (4.31) is the cumulant transform of a one-dimensional gamma convolution; see, for instance, [6]. Therefore, process $R_t(u)$ follows the distribution of a univariate time changed Brownian motion B_{T_t}, in which the Lévy subordinator T_t is such that T_1 follows a univariate generalized gamma convolution. There exists a strong link between random variables distributed as positive Generalized Gamma Convolutions and so-called Wiener-Gamma integrals; see, e.g., [19] for a survey. Let $(\gamma_t, t\geq 0)$ be a standard Gamma process and $h : \mathbb{R}_+ \to \mathbb{R}_+$ a Borel function such that $\int_0^\infty \log(1+h(u))du < \infty$. The random variable $\tilde{\Gamma}(h)$ defined as

$$\tilde{\Gamma}(h) := \int_0^\infty h(s)d\gamma_s \tag{4.32}$$

is called a Wiener-Gamma integral. Therefore, $\tilde{\Gamma}(h)$ can be written as the limit of linear combinations of independent gamma random variables, with weights given by the values of function h. This feature implies that Generalized Gamma Convolution subordinators can be written as Wiener-Gamma integrals, leading to a natural extension of Gamma subordinators.

Overall, we obtain that $R_t(u)$ follows the same distribution of a univariate Brownian motion time-changed by a Wiener-Gamma integral process. In this sense, we obtain a natural multivariate extension of the univariate Variance Gamma process.

4.3.2.2 Marginal Distributions: Tempered Subordinators

The specific form of function (4.26) implied by tempered matrix subordinators is addressed next.

Corollary 4.2. *Let* $X \sim G_\beta(I_n)$, $0 < \beta < 1$. *It then follows:*

$$
\begin{aligned}
-\frac{1}{\Delta}\ln \Phi(u,\Delta) &= -k_\beta \left\{ \int\limits_{\mathbf{S}^n \cap \mathcal{S}_n^+} \ln\left(1+\frac{1}{2}tr(uu^{'}V)\right)^\beta dV - c_n \right\} \\
&= -k_\beta \left\{ \int\limits_0^\infty \ln\left(1+\frac{1}{2}v\right)^\beta \nu_{uu'}(dv) - c_n \right\}, \qquad (4.33)
\end{aligned}
$$

where $\nu_{uu'}(B) := \int_{\mathbf{S}^n \cap \mathcal{S}_n^+} I_B(tr(uu^{'}V))\nu_V(dV)$ *is the measure on* $(0,\infty)$ *induced trough the transformation* $V \to tr(uu^{'}V)$, *and* k_β, c_n *are defined in Corollary 4.3.*

It follows that $-\frac{1}{\Delta}\ln\Phi(u,\Delta)$ is the cumulant transform of a one-dimensional tempered β-stable distribution; see [26]. Therefore, $R_t(u)$ follows the same distribution of a one-dimensional Brownian motion time-changed by a one-dimensional tempered β-stable subordinator. When $\beta = 1/2$, the subordinator is Inverse Gaussian distributed, and $R_t(u)$ follows a Normal Inverse Gaussian process. Therefore, we obtain a natural multivariate extension of univariate Normal Inverse Gaussian processes.

4.3.3 Compound Poisson Matrix Subordinators

Matrix compound processes with positive jumps are multivariate compound Poisson processes taking values in the cone of positive definite matrices and having positive definite jumps. Their Lévy density is the product of a constant jump intensity $\lambda > 0$ and a density $g(X)$, defined on $\mathcal{S}_n^+$, for the jump size. Without loss of generality, it is possible to decompose $g(X)$ as $g(X) = \tilde{g}(r)\Gamma(V)$, for a spectral density Γ, defined on $\mathbf{S}^n \in \mathcal{S}_n^+$, and a second density $\tilde{g}$ defined on $\mathbb{R}^+$. The cumulant transform of matrix compound processes X takes the form

$$\mathcal{K}_X(\Theta) = \int\limits_{\mathbf{S}^n \cap \mathcal{S}_n^+} \lambda \int\limits_0^\infty (1 - e^{-rtr(\Theta V)}) \tilde{g}(dr) \Gamma(dV). \tag{4.34}$$

The integral with respect to dr in equation (4.34) can be interpreted as the cumulant transform of a univariate Compound Poisson process with jump-intensity λ and jump size density $\tilde{g}$, evaluated in $tr(\Theta V)$. Therefore, $\mathcal{K}_X(\Theta)$ can be interpreted as an average of cumulant transforms of standard univariate Compound Poisson processes, each weighted by spectral density $\Gamma(dV)$. The specific form of function (4.26) for matrix compound subordinators is as follows.

Corollary 4.3. *Let $(X_t)_{t \geq 0}$ be a compound Poisson matrix subordinator. It then follows:*

$$\begin{aligned} -\frac{1}{\Delta} \ln \Phi(u, \Delta) &= \int\limits_{\mathcal{S}^n \cap \mathcal{S}_n^+} \lambda \int\limits_0^\infty \left(1 - e^{-\frac{r}{2} tr\left((uu' V)\right)}\right) \tilde{g}(dr) \Gamma(dV) \\ &= \int\limits_{\mathcal{S}^n \cap \mathcal{S}_n^+} \lambda \left(1 - \phi_r\left(-\frac{1}{2} tr\left(uu' V\right)\right)\right) \Gamma(dV). \end{aligned} \tag{4.35}$$

where $\phi_r(\cdot)$ is the Laplace transform of the univariate density $\tilde{g}$.

4.4 Specification Analysis

We address in more detail the issue of specifying portfolio time changes and their dependence with Lévy process L_t^u in equation (4.3), in order to develop models implying multivariate leverage effects. A convenient framework in this respect is given by the class of matrix AJD, introduced in [20], which can be used to specify portfolio business activities correlated with return shocks, while preserving a good degree of model tractability. We study in detail an explicit model for the diffusion case and show how to compute the leverage-neutral measure and the characteristic function of the corresponding multivariate return process. The next assumption fixes the relevant setting for this section.

Assumption 4.5. *$W_t^u := (W_{1t}^u, W_{2t}^u)$ is a family of standard Brownian motions in $\mathbb{R}^2$ indexed by $u \in \mathbb{R}^2$. X_t^u is a family of symmetric positive definite matrix processes identically distributed as X_t, indexed by $u \in \mathbb{R}^2$. Let $T_{it}^u := \int_0^t tr(uu'(V_s^{ui})(V_s^{ui})')ds$, where V_t^u is the* *unique symmetric positive definite square root of* X_t^u. $R_t(u)$ *is specified as the portfolio time-changed Brownian motion:*

$$R_t(u) = W^u_{1T^u_{1t}} + W^u_{2T^u_{2t}} \tag{4.36}$$

Under Assumption 4.5, function $\Phi(u, \Delta, x)$ in Theorem 4.1 reads:

$$\Phi(u, \Delta, x) = \mathbb{E}^{\mathbb{Q}(u)}\left[\exp\left(-\frac{1}{2}(T^1_\Delta + T^2_\Delta)\right)\right] = \mathcal{L}^{\mathbb{Q}(u)}_{\int_0^\Delta X^u_s ds}\left(-\frac{1}{2}uu', \Delta, x\right) \tag{4.37}$$

where leverage-neutral measure $\mathbb{Q}(u)$ has density with respect to $\mathbb{P}$ given by:

$$\frac{d\mathbb{Q}(u)}{d\mathbb{P}}\Big|_{\mathcal{F}_t} = \exp\left[i(W^u_{1T^u_{1t}} + W^u_{2T^u_{2t}}) + \frac{1}{2}tr\left(uu'\int_0^t X^u_s ds\right)\right] \tag{4.38}$$

and $\mathcal{L}^{\mathbb{Q}(u)}_{\int_0^\Delta X^u_s ds}(\cdot, \Delta, x)$ is the leverage-neutral conditional Laplace transform of integrated process $(X^u_t)_{t\geq 0}$.

Overall, we see that in the context of Assumption 4.5 analytical tractability of function $\Phi(u, \Delta, x)$ is granted as soon as the leverage neutral Laplace transform of integrated process $(X^u_t)_{t\geq 0}$ is available in closed form.

4.4.1 Affine Portfolio Activity Rates

Let Ω, M, Q be 2×2 parameter matrices and $\{(B^u_t)_{t\geq 0} : u \in \mathbb{R}^2\}$ be a family of 2×2 matrices of standard Brownian motions. X^u_t is a matrix AJD if it follows the (matrix) stochastic differential equation:

$$dX^u_t = (\Omega\Omega' + MX^u_t + X^u_t M')dt + \sqrt{X_t}dB^u_t Q + Q' dB^{u\prime}_t \sqrt{X^u_t} + dJ^u_t, \tag{4.39}$$

with initial conditions $X^u_0 = x \in \mathcal{S}^+_n$, where J^u_t is a pure jump process with values in $\mathcal{S}^+_n$ identically distributed across $u \in \mathbb{R}^2$. Jumps are realized with an intensity $\lambda_X(X^u_t) := \lambda_{X,0} + tr(\lambda_{X,1}X^u_t)$, such that $\lambda_{X,0} \geq 0$ and $\lambda_{X,1} \in \mathcal{S}^+_n$.[7]

Since jump size and arrival intensity are separately described, X^u_t is a matrix-valued compound Poisson process featuring finite jump activity. For $\Omega\Omega' \gg Q'Q$ symmetric matrix X^u_t is positive semidefinite, which gives rise to well-defined portfolio activity rates, specified as $tr(uu'X^u_t)$ in Assumption 4.5. Note that since Brownian motions W^u_t and B^u_t can be correlated, under Assumption 4.5 we can use the dynamics (4.39) to specify different types of multivariate return processes featuring leverage effects.

[7] Technical conditions for equation (4.39) to have a strong solution are provided, e.g., in [7].

The Laplace transform of the integrated process X_t^u is exponentially affine and is characterized as follows.

Proposition 4.2. *Given the affine dynamics (4.39) for matrix process X_t^u, the Laplace transform of the integrated process $\int_0^{\Delta} X_t^u dt$ is exponentially affine:*

$$E\left[\exp\left(tr\left(-\frac{1}{2}uu'\int_0^{\Delta} X_t^u dt\right)\right)\Big|X_0^u = x\right] = \exp(B^u(\Delta) + tr(A^u(\Delta)x), \quad (4.40)$$

with functions $B^u(\Delta) \in \mathbb{R}$ and $A^u(\Delta) \in \mathcal{S}_n^+$ that solve the following system of matrix Riccati equations:

$$\frac{dA^u(\Delta)}{d\Delta} = -\frac{1}{2}uu' + M'A^u(\Delta) + A^u(\Delta)M + 2A^u(\Delta)Q'QA^u(\Delta) \quad (4.41)$$

$$+\lambda_{X,1}[\Theta^X(A^u(\Delta)) - 1] \ , \quad (4.42)$$

$$\frac{dB^u(\Delta)}{d\Delta} = tr(A^u(\Delta)\Omega\Omega') + \lambda_{X,0}[\Theta^X(A^u(\Delta)) - 1] \ , \quad (4.43)$$

where Θ^X is the Laplace transform of jump size J, subject to terminal condition $B^u(0) = 0$ and $A^u(0) = 0$.

Closed form solutions for the coefficients of the above matrix Riccati equations can be derived if $\Omega\Omega' = \beta Q'Q$ for some $\beta > 1$ and if $\lambda_{X,1} = 0$, i.e., jump intensities are constant; see again [20].

4.4.2 Multivariate Leverage Effects Through Matrix Diffusions

Consider the state dynamics (4.39) for matrix process X_t^u in the pure diffusion case where $\lambda_X(X_t^u) = 0$. We can specify leverage effects in our multivariate model, by correlating the Brownian motions W_t^u and B_t^u driving return and portfolio time-change shocks. It is convenient to specify a linear correlation structure, as shown in the next assumption.

Assumption 4.6. $\{(W_t^u)_{t\geq 0} := (W_{1t}^u, W_{2t}^u)'_{t\geq 0} : u \in \mathbb{R}^2\}$ *is a family of bivariate standard Brownian motions given by:*

$$W_t^u = B_t^u\rho + \sqrt{1-\rho'\rho}Z_t^u \ , \quad (4.44)$$

where $\{Z_t^u : u \in \mathbb{R}^2\}$ is another family of bivariate standard Brownian motions, independent of B_t^u, and $\rho \in \mathbb{R}^2$ is a fixed correlation vector such that $\rho'\rho \leq 1$.

Preservation of the affine structure of process X_t^u under the leverage-neutral measure is in general not granted. This feature depends on the assumptions about the correlation structure between Lévy shocks and business times, as well as the specification of the multivariate subordination procedure defining returns. The next proposition shows that Assumptions 4.5 and 4.6 together indeed imply a matrix AJD for X_t^u under leverage neutral measure $\mathbb{Q}(u)$. To the best of our knowledge, this is the first multivariate result of this kind in the literature.

Proposition 4.3. *Given Assumptions 4.5 and 4.6, the dynamics of X_t^u under leverage neutral measure $\mathbb{Q}(u)$ is:*

$$dX_t^u = (\Omega\Omega' + \tilde{M}X_t^u + X_t^u\tilde{M}')dt + \sqrt{X_t^u}d\tilde{B}_t^u Q + Q'd\tilde{B}'^u_t\sqrt{X_t^u}, \tag{4.45}$$

where $\tilde{B}_t$ is a 2×2 matrix of $\mathbb{Q}(u)-$Brownian motions and $\tilde{M} := M + i(u\rho'Q)'$.

Proof. In order to apply Girsanov's theorem, we rewrite the Radon-Nikodym density given in equation (4.22) as

$$\left.\frac{d\mathbb{Q}(u)}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \mathcal{E}(iW_{T_{1t}^u})\mathcal{E}(iW_{T_{2t}^u}) \tag{4.46}$$

where $\mathcal{E}(\cdot)$ denotes the stochastic exponential. The transformation of the $\mathbb{P}$-Brownian motion B_t^u into $\mathbb{Q}(u)$-Brownian motion $\tilde{B}_t^u$ can be done by a slight extension of Girsanov's theorem to complex-valued measures; see [11].

$$d\tilde{B}^{kju} = dB^{kju} - \left[dB^{kju}, i\Big(\sqrt{tr(uu'V^{u1}V^{u1\prime})}dW_1^u + \sqrt{tr(uu'V^{u2}V^{u2\prime})}dW_2^u\Big)\right] =$$

$$dB^{kju} - \left[dB^{kju}, i\Big(\sqrt{tr(uu'V^{u1}V^{u1\prime})}(\rho_1 dB^{11u} + \rho_2 dB^{12u}) + \sqrt{tr(uu'V^{u2}V^{u2\prime})}(\rho_1 dB^{21u} + \rho_2 dB^{22u})\Big)\right],$$

which implies $d\tilde{B}_t = dB_t^u - iA_t dt$, with 2×2 matrix A_t given by:

$$A_t := \begin{pmatrix} \sqrt{tr(uu'V_{1t}^u V_{1t}^{u\prime})} \\ \sqrt{tr(uu'V_{2t}^u V_{2t}^{u\prime})} \end{pmatrix}\rho' = \begin{pmatrix} u_1 V_t^{11} + u_2 V_t^{12} \\ u_1 V_t^{21} + u_2 V_t^{22} \end{pmatrix}\rho' = \sqrt{X_t^u}u\rho' \,. \tag{4.47}$$

Substituting $dB_t^u = d\tilde{B}_t^u + iA_t dt$ in equation (4.39) and recalling that $\lambda_X(X_t^u) = 0$, the dynamics of X_t^u under complex measure $\mathbb{Q}(u)$ is:

$$dX_t^u = (\Omega\Omega' + \tilde{M}X_t^u + X_t^u\tilde{M}')dt + \sqrt{X_t^u}d\tilde{B}_t^u Q + Q'd\tilde{B}'^u_t\sqrt{X_t^u}, \tag{4.48}$$

where $\tilde{B}^u$ is a 2×2 matrix of standard $\mathbb{Q}(u)-$ Brownian motions parameterized by u and $\tilde{M} := M + i(u\rho'Q)'$. This concludes the proof. $\square$

The proof of Proposition 4.3 explicitly shows whether or not the affine structure of X_t^u dynamics under leverage neutral measure $\mathbb{Q}(u)$ is preserved, given a specification of the chosen time-subordination approach. Obviously, Assumption 4.5 is not the only potential way in which portfolio time-changes can be specified using matrix AJD process X_t^u. For instance, the class of matrix AJD activity rates of the form $v_t := tr(H_i X_t^u)$ leads to well-defined portfolio time-changes, given symmetric positive semi-definite matrices H_i, $i = 1, \ldots, d$. However, such a portfolio time change does not preserve an affine drift under leverage neutral measure $\mathbb{Q}(u)$ when Assumption 4.6 is applied. This important fact can be verified from the proof of Proposition 4.3, by noting that in that case the resulting matrix A_t in equation (4.47) cannot be rewritten as a multiple of $\sqrt{X_t}$.

The affine structure of X_t^u under leverage neutral measure $\mathbb{Q}(u)$, implied by Proposition 4.3, allows us to derive simple analytical formulas for function $\Phi(u, \Delta, x)$ also in the presence of leverage effects specified according to Assumptions 4.5 and 4.6. The detailed result is presented in the next proposition.

Proposition 4.4. *Under Assumption 4.6, we have:*

$$\Phi(u, \Delta, x) = \mathbb{E}^{\mathbb{Q}(u)}\left[\exp\left(-\frac{1}{2}tr(\int_0^{\Delta} uu'X_s^u ds)\right)\Big|X_0^u = x\right] \tag{4.49}$$

$$= \exp((B^u(\Delta) + tr(A^u(\Delta)x)) \tag{4.50}$$

where functions $A^u(\Delta)$, $B^u(\Delta)$ solve the following system of matrix Riccati equations with complex-valued coefficients:

$$\frac{dA^u(\Delta)}{d\Delta} = -\frac{1}{2}uu' + (M + iQ'\rho u')'A^u(\Delta) + A^u(\Delta)(M + iQ'\rho u') \tag{4.51}$$

$$+2A^u(\Delta)Q'QA^u(\Delta) \ , \tag{4.52}$$

$$\frac{dB^u(\Delta)}{d\Delta} = \beta tr(A^u(\Delta)Q'Q) \ , \tag{4.53}$$

subject to terminal condition $A^u(0) = 0$ and $B^u(0) = 0$. For $\Omega\Omega' = \beta Q'Q$, the closed form solution of these equations is $A^u(\Delta) = C_{22}(\Delta)^{-1}C_{21}(\Delta)$, where $C_{ij}(\Delta)$ is the 2×2 block of the matrix exponential,

$$\begin{pmatrix} C_{11}(\Delta) \; C_{12}(\Delta) \\ C_{21}(\Delta) \; C_{22}(\Delta) \end{pmatrix} := \exp\left(\Delta\begin{pmatrix} M + i(u\rho'Q)' & -2Q'Q \\ -\frac{1}{2}uu' & -(M' + iu\rho'Q) \end{pmatrix}\right) \tag{4.54}$$

and

$$B^u(\Delta) = -\frac{1}{2}tr\left[\beta\log(C_{22}(\Delta) - \tau(M' + iu\rho'Q))\right] \ . \tag{4.55}$$

In the context of Proposition 4.4, derivative pricing can be efficiently performed by transform methods for a broad class of models. Concrete examples and implementation are left for future research.

4.5 Conclusions

We propose a new family of multivariate time-changed Lévy processes to model multivariate sources of risk in finance. Using powerful time-change techniques, our approach allows us to address several well-known characteristics of financial returns in a multivariate context, including joint non-normal behavior, stochastic volatilities and correlations among different assets, multivariate leverage effects or self-exciting behavior. This framework can include as special cases a variety of models in the literature, gives rise to several new multivariate models and it is similarly simple to apply using the characteristic function methodology as in the univariate context. We specify the distribution of our multivariate time-changed Lévy processes based on parametric families of portfolio time-changes driven by positive matrix state processes. This matrix subordination procedure allows us to extend in a natural way univariate Variance Gamma and Normal Inverse Gaussian models to a new class of multivariate Lévy processes. In order to introduce multivariate leverage effects between returns, volatilities and correlations, we follow the standard approach of correlating shocks in the Lévy processes driving returns and the increments of the underlying multivariate time change. We show that, as in the univariate case, the mathematical complexity introduced by multivariate leverage effects can be neutralized trough an appropriate complex-valued measure change, leading to returns characteristic functions that satisfy a bond-type pricing formula under an appropriate leverage-neutral measure. Finally, in the context of matrix affine jump diffusions, we propose concrete specifications of portfolio time-changes that are convenient in order to preserve affine transform solutions in connection with affine leverage structures. In these settings, derivative pricing can be performed efficiently for a broad class of contingent claims, using transform and Fast Fourier Transform methods, similar to several univariate affine settings.

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Part III

Appendix

Declaration

,,The work submitted in this dissertation is the result of my own and coauthors' investigation, except where otherwise stated. It has not already been accepted for any degree, and is also not being concurrently submitted for any other degree."

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